

Question 8

A transformation T is performed in this order:

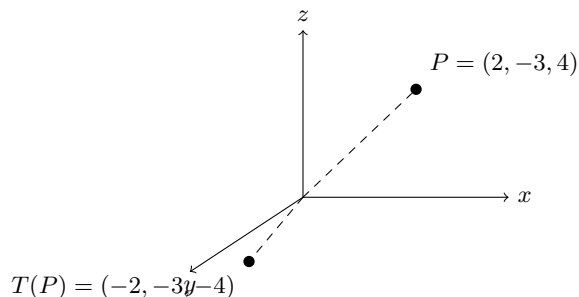
1. reflect across the yz -plane;
2. then reflect the result across the xy -plane.

Tasks

1. Find a coordinate formula for $T(x, y, z)$.
2. Compute $T(2, -3, 4)$.
3. Identify the single rigid motion equivalent to this composition.
4. Prove directly from the three-dimensional distance formula that T preserves the distance between every pair of points.

Question 8 – Solution

Strategy Track which coordinate changes sign under each reflection. Then compare squared distances before and after the transformation.



Coordinate rule Reflection across the yz -plane changes x to $-x$; reflection across the xy -plane changes z to $-z$. Therefore

$$\boxed{T(x, y, z) = (-x, y, -z)}, \quad T(2, -3, 4) = \boxed{(-2, -3, -4)}.$$

Geometric interpretation The y -coordinate is fixed while both perpendicular coordinates reverse. This is a rotation through 180° about the y -axis.

Distance preservation For $P = (x_1, y_1, z_1)$ and $Q = (x_2, y_2, z_2)$,

$$d(TP, TQ)^2 = (-x_1 + x_2)^2 + (y_1 - y_2)^2 + (-z_1 + z_2)^2 = d(P, Q)^2.$$

Both distances are nonnegative, so they are equal.

Verification Applying T twice restores every coordinate, as expected for a half-turn.