

Question 7

Find every sphere satisfying all of the following conditions:

1. The sphere lies entirely in the octant

$$x \geq 0, \quad y \leq 0, \quad z \geq 0.$$

2. It is tangent to each of the three coordinate planes.

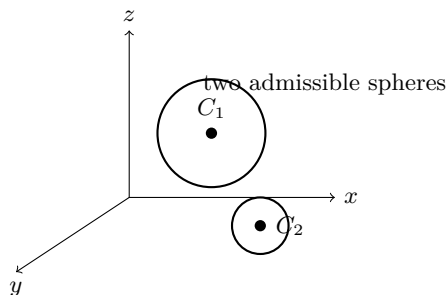
3. It passes through the point

$$P = (1, -2, 3).$$

For each sphere, state its center, radius, and Cartesian equation. Then give a geometric explanation for why two different spheres satisfy the same conditions.

Question 7 – Solution

Strategy Tangency to all three coordinate planes makes the center's coordinate magnitudes equal to the radius. The specified octant fixes their signs.



Center condition If the radius is $r > 0$, then the center must be

$$C = (r, -r, r).$$

Since P lies on the sphere,

$$(1 - r)^2 + (-2 + r)^2 + (3 - r)^2 = r^2.$$

Expanding gives $2r^2 - 12r + 14 = 0$, or

$$r^2 - 6r + 7 = 0.$$

Hence $r = 3 \pm \sqrt{2}$, both positive.

Results The two spheres are

$$(x - r)^2 + (y + r)^2 + (z - r)^2 = r^2, \quad r = 3 \pm \sqrt{2}.$$

Why two? Along the line of possible centers $(r, -r, r)$, the distance to P first decreases and then increases, so the equation “distance to center equals r ” can be met at two scales. Direct substitution verifies both roots, and each sphere has extreme coordinates touching—but not crossing—the three coordinate planes.