

**Question 6**

Two points determine a directed line segment:

$$A = (-4, 1, 7), \quad B = (6, -3, -1).$$

**Tasks**

1. Find the point  $P$  that divides  $\overline{AB}$  internally in the ratio

$$AP : PB = 2 : 3.$$

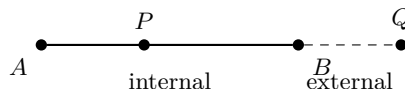
2. Find the point  $Q$  that divides the same line externally in the ratio

$$AQ : QB = 2 : 3.$$

3. Verify both ratios using three-dimensional distances, and distinguish clearly between internal and external division.

## Question 6 – Solution

**Strategy** Parameterize the line by  $X(t) = A + t(B - A)$ . Internal division has  $0 < t < 1$ ; external division requires  $t$  outside that interval.



**Internal point** Since  $B - A = (10, -4, -8)$  and  $AP/AB = 2/(2 + 3) = 2/5$ ,

$$P = A + \frac{2}{5}(B - A) = (-4, 1, 7) + (4, -8/5, -16/5) = \boxed{(0, -3/5, 19/5)}.$$

**External point** For  $Q = A + t(B - A)$  beyond  $A$ , take  $t < 0$ . The ratio is

$$\frac{AQ}{QB} = \frac{-t}{1-t} = \frac{2}{3},$$

which gives  $t = -2$ . Thus

$$Q = A - 2(B - A) = \boxed{(-24, 9, 23)}.$$

**Verification** Because  $P - A = (2/5)(B - A)$  and  $B - P = (3/5)(B - A)$ , their lengths have ratio  $2 : 3$ . Also  $Q - A = -2(B - A)$  and  $B - Q = 3(B - A)$ , again giving  $AQ : QB = 2 : 3$ .