

Question 4

Let S be the set of points whose distance to the xy -plane is no greater than their distance to the z -axis:

$$S = \{P : d(P, xy\text{-plane}) \leq d(P, z\text{-axis})\}.$$

Tasks

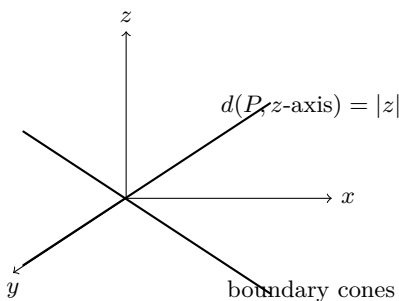
1. Derive a Cartesian inequality describing S .
2. Describe the boundary surface of S .
3. Determine whether each test point belongs to S :

$$P_1 = (1, 2, 3), \quad P_2 = (2, -2, 1).$$

4. Describe the reflectional and rotational symmetries of S .

Question 4 – Solution

Strategy Translate each geometric distance into coordinates: distance to the xy -plane is $|z|$, while distance to the z -axis is the radial distance $\sqrt{x^2 + y^2}$.



Inequality and boundary We require

$$|z| \leq \sqrt{x^2 + y^2} \iff \boxed{z^2 \leq x^2 + y^2}.$$

The boundary $z^2 = x^2 + y^2$ is a double cone with vertex at the origin. The set S is the region on or outside that cone in the radial direction.

Membership For $(1, 2, 3)$, $z^2 = 9 > 1^2 + 2^2 = 5$, so it is not in S . For $(2, -2, 1)$, $z^2 = 1 \leq 8$, so it is in S .

Symmetry and verification The inequality uses only squares, so changing any coordinate's sign preserves membership; rotations about the z -axis also preserve $x^2 + y^2$. On the xy -plane, every point satisfies it, while nonzero points on the z -axis do not, matching the distance interpretation.