

Question 3

Consider the fixed points

$$A = (-2, 0, 0), \quad B = (4, 0, 0),$$

and let $P = (x, y, z)$ satisfy the distance condition

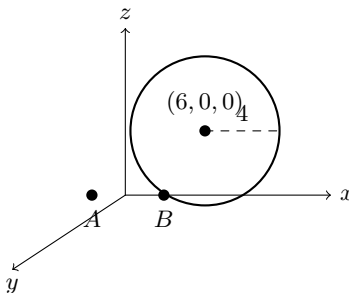
$$d(P, A) = 2d(P, B).$$

Tasks

1. Derive a Cartesian equation for the locus of all possible points P .
2. Identify the surface by stating its center and radius.
3. Determine which of A and B lies inside the surface.
4. Explain geometrically what the factor 2 requires of points on the locus.

Question 3 – Solution

Strategy Square the distance relation, expand, and complete the square. The resulting surface is an Apollonius sphere.



Derivation The condition is

$$(x + 2)^2 + y^2 + z^2 = 4((x - 4)^2 + y^2 + z^2).$$

After collecting terms and dividing by 3,

$$x^2 - 12x + y^2 + z^2 + 20 = 0.$$

Completing the square gives

$$\boxed{(x - 6)^2 + y^2 + z^2 = 16}.$$

Thus the locus is the sphere centered at $(6, 0, 0)$ with radius 4.

Interpretation Point B is 2 units from the center and lies inside; A is 8 units from the center and lies outside. A point must remain closer to B because its distance to A is required to be twice as large.

Verification At the axial endpoints $(2, 0, 0)$ and $(10, 0, 0)$, the distance pairs are $(4, 2)$ and $(12, 6)$, respectively, confirming the ratio.