

Question 2

A sphere passes through four specified points in three-dimensional space.

Given

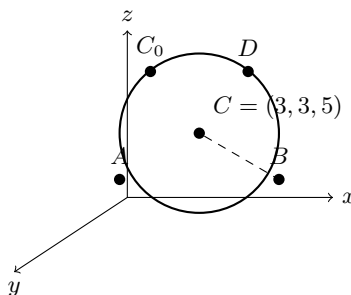
$$\begin{aligned}A &= (1, 0, 2), & B &= (5, 0, 2), \\C &= (1, 6, 2), & D &= (1, 0, 8).\end{aligned}$$

Tasks

1. Find the center and radius of the sphere.
2. Write its Cartesian equation.
3. Explain why the four given points determine a unique sphere.
4. Verify that all four points satisfy your equation.

Question 2 – Solution

Strategy A sphere's center is equidistant from all four points. Perpendicular-bisector conditions can be obtained by equating squared distances.



Center Let the center be $K = (h, k, \ell)$. From $d(A, K)^2 = d(B, K)^2$,

$$(h - 1)^2 + k^2 + (\ell - 2)^2 = (h - 5)^2 + k^2 + (\ell - 2)^2,$$

so $h = 3$. Comparing A with C gives $k = 3$, and comparing A with D gives $\ell = 5$. Hence the center is $(3, 3, 5)$ and

$$r^2 = (3 - 1)^2 + (3 - 0)^2 + (5 - 2)^2 = 22.$$

Therefore

$$\boxed{(x - 3)^2 + (y - 3)^2 + (z - 5)^2 = 22}.$$

Uniqueness and verification The three difference vectors from A point independently in the x -, y -, and z -directions, so their perpendicular-bisector planes meet in exactly one point. Substitution shows that each given point produces squared distance 22.