

Question 1

A point $P = (x, y, z)$ is known only through its octant and its distances to the coordinate axes.

Given

$$x < 0, \quad y > 0, \quad z < 0,$$

$$d(P, x\text{-axis}) = \sqrt{41},$$

$$d(P, y\text{-axis}) = \sqrt{34},$$

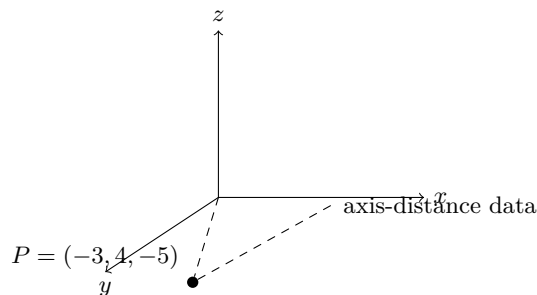
$$d(P, z\text{-axis}) = 5.$$

Tasks

1. Determine the coordinates of P .
2. Verify your result using all three axis distances.
3. Explain why only two of the three distance measurements would not determine P uniquely.

Question 1 – Solution

Strategy Squared distance to an axis is the sum of the squares of the other two coordinates. Solve first for x^2, y^2, z^2 , then use the octant to determine signs.



Equations The data give

$$y^2 + z^2 = 41, \quad x^2 + z^2 = 34, \quad x^2 + y^2 = 25.$$

Adding the last two equations and subtracting the first yields $2x^2 = 18$, so $x^2 = 9$. Similarly,

$$2y^2 = 41 + 25 - 34 = 32, \quad 2z^2 = 41 + 34 - 25 = 50.$$

Thus $|x| = 3$, $|y| = 4$, and $|z| = 5$. The stated octant fixes the signs:

$$\boxed{P = (-3, 4, -5)}.$$

Verification The resulting axis distances are

$$\sqrt{4^2 + (-5)^2} = \sqrt{41}, \quad \sqrt{(-3)^2 + (-5)^2} = \sqrt{34}, \quad \sqrt{(-3)^2 + 4^2} = 5.$$

With only two distances, one squared coordinate would remain free, so the third is essential.