

Question 1

Find the domain of

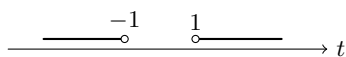
$$\vec{r}(t) = \left\langle \sqrt{t+2}, \ln(1-t), \frac{1}{t^2-1} \right\rangle.$$

Evaluate $\vec{r}(-1/2)$ and determine whether $\lim_{t \rightarrow -1^+} \vec{r}(t)$ exists in $\mathbb{R}^3$.

Question 1 – Solution

Strategy Intersect the domains of all three scalar components; vector limits exist exactly when every component limit is finite.

common component domain



Domain The restrictions are $t \geq -2$, $t < 1$, and $t \neq \pm 1$. Hence

$$[-2, -1) \cup (-1, 1).$$

Also $\vec{r}(-1/2) = \langle \sqrt{3/2}, \ln(3/2), -4/3 \rangle$.

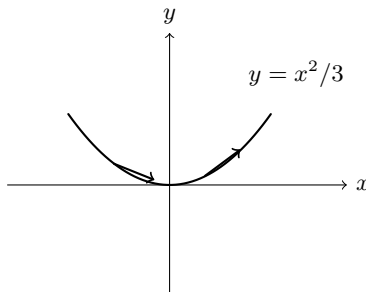
Limit As $t \rightarrow -1^+$, $1/(t^2 - 1) \rightarrow -\infty$, so no finite vector limit exists in $\mathbb{R}^3$.

Question 2

For $\vec{r}(t) = \langle 3t, t^2, 2 - t \rangle$, eliminate the parameter to describe the curve as the intersection of two surfaces. State its orientation as t increases and locate the point corresponding to $t = -1$.

Question 2 – Solution

Strategy Solve the simplest component for t and substitute into the others.



Elimination Since $x = 3t$, $t = x/3$. Therefore

$$\boxed{y = x^2/9, \quad z = 2 - x/3}.$$

The curve is the intersection of a parabolic cylinder and a plane.

Orientation and point As t increases, x increases and z decreases. At $t = -1$, the point is $\boxed{(-3, 1, 3)}$.

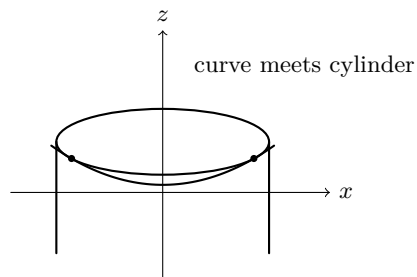
Verification Substituting these coordinates into both surface equations confirms membership.

Question 3

The curve $\vec{r}(t) = \langle t, t^2, 2t - 1 \rangle$ meets the cylinder $x^2 + y^2 = 2$. Find every intersection point and the corresponding parameter values.

Question 3 – Solution

Strategy Substitute the curve's coordinate functions into the surface equation.



Parameter equation We need $t^2 + t^4 = 2$. Let $u = t^2 \geq 0$; then $u^2 + u - 2 = 0$, so $(u - 1)(u + 2) = 0$. Thus $t^2 = 1$ and $t = \pm 1$.

Points At $t = 1$, $\vec{r} = (1, 1, 1)$; at $t = -1$, $\vec{r} = (-1, 1, -3)$. Hence the intersections are

$$\boxed{(1, 1, 1) \quad \text{and} \quad (-1, 1, -3)}.$$

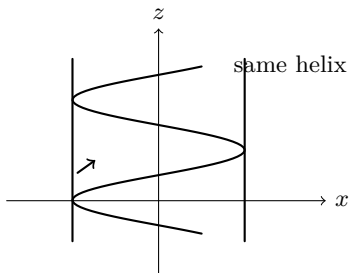
Verification Both points satisfy $x^2 + y^2 = 2$.

Question 4

Show that $\vec{r}_1(t) = \langle \cos t, \sin t, t \rangle$ and $\vec{r}_2(u) = \langle \cos(2u), \sin(2u), 2u \rangle$ have the same image. Compare their orientations and speeds of traversal at matching points. Then give a reparameterization with the opposite orientation.

Question 4 – Solution

Strategy Relate parameters by matching all three components.



Same image Setting $t = 2u$ makes $\vec{r}_1(t) = \vec{r}_2(u)$, so both trace the same helix. As each parameter increases, z increases, so orientations agree.

Traversal rate A change Δu produces twice the angular and vertical change of the same Δt ; equivalently, $\vec{r}_2(u) = \vec{r}_1(2u)$ traverses the image twice as fast with respect to its parameter.

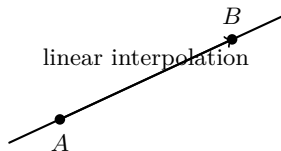
Reverse orientation For example, $\boxed{\vec{q}(s) = \langle \cos s, -\sin s, -s \rangle = \vec{r}_1(-s)}$.

Question 5

Construct a vector function that travels from $A = (-2, 1, 4)$ to $B = (3, -1, 0)$ during $0 \leq t \leq 1$, but does so nonuniformly: it starts and ends at rest while remaining on the line segment.

Question 5 – Solution

Strategy Use $\vec{r} = A + s(t)(B - A)$ with a scalar progress function satisfying $s(0) = 0$, $s(1) = 1$, and $s'(0) = s'(1) = 0$.



Progress function The cubic $s(t) = 3t^2 - 2t^3$ has the required endpoint values and zero endpoint derivatives. Since $0 \leq s(t) \leq 1$ on $[0, 1]$, the path stays on the segment.

Vector function Because $B - A = \langle 5, -2, -4 \rangle$,

$$\boxed{\vec{r}(t) = \langle -2, 1, 4 \rangle + (3t^2 - 2t^3) \langle 5, -2, -4 \rangle}, \quad 0 \leq t \leq 1.$$

Verification Substitution gives the correct endpoints, and $\vec{r}'(0) = \vec{r}'(1) = \vec{0}$.

Question 6

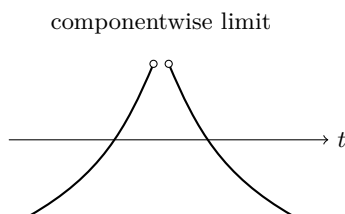
Find constants a, b, c so that

$$\lim_{t \rightarrow 0} \left\langle \frac{\sqrt{1+at}-1}{t}, \frac{\sin(bt)}{t}, \frac{e^{ct}-1}{t} \right\rangle = \langle 2, 3, -4 \rangle.$$

Justify the limit component by component.

Question 6 – Solution

Strategy A vector limit equals the target precisely when all component limits equal their corresponding target components.



First component Rationalizing gives

$$\frac{\sqrt{1+at}-1}{t} = \frac{a}{\sqrt{1+at}+1} \rightarrow \frac{a}{2},$$

so $a = 4$.

Other components Standard limits give $\sin(bt)/t \rightarrow b$ and $(e^{ct}-1)/t \rightarrow c$. Hence $a = 4, b = 3, c = -4$.

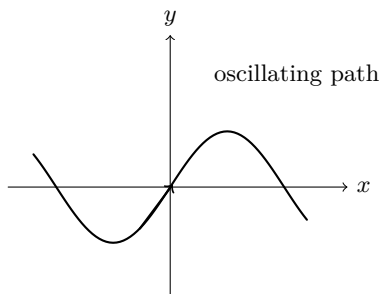
Verification The three component limits are respectively $2, 3, -4$.

Question 7

Analyze the planar vector function $\vec{r}(t) = \langle t - \sin t, 1 - \cos t \rangle$ for $0 \leq t \leq 4\pi$. Find the points for $t = 0, \pi, 2\pi, 3\pi, 4\pi$, identify repeated heights, and describe the geometric motion.

Question 7 – Solution

Strategy Evaluate at multiples of π and use the periodicity of sine and cosine.



Key points The points are

$$(0,0), (\pi,2), (2\pi,0), (3\pi,2), (4\pi,0).$$

The height oscillates between 0 and 2 while $x = t - \sin t$ increases from 0 to 4π .

Geometry The curve is two arches of a cycloid, moving left to right, with cusps at $(0,0), (2\pi,0), (4\pi,0)$.

Verification Since $1 - \cos t \in [0, 2]$ and $x'(t) = 1 - \cos t \geq 0$, the description is consistent.

Question 8

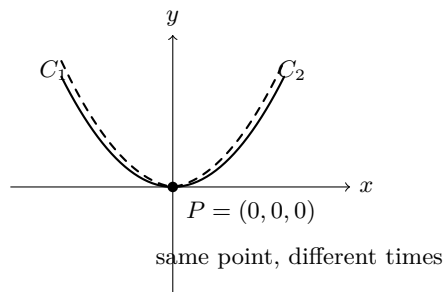
Curves C_1 and C_2 are given by

$$\vec{r}_1(t) = \langle t, t^2, t^3 \rangle, \quad \vec{r}_2(s) = \langle 1 - s, (1 - s)^2, s - 1 \rangle.$$

Determine all geometric intersection points. If particles use their displayed parameters as time, determine whether they collide.

Question 8 – Solution

Strategy Geometric intersection permits $t \neq s$; collision requires $t = s$ as well as equal positions.



Intersection From the first coordinate, set $u = 1 - s = t$. The second coordinates then agree automatically. The third coordinates require $t^3 = -u = -t$, so $t(t^2 + 1) = 0$. Thus $t = 0$ and $s = 1$, giving the sole intersection $\boxed{(0, 0, 0)}$.

Collision The required times differ, so particles using the displayed parameters do not collide.

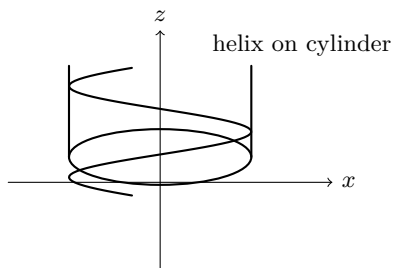
Check At $t = 0$, $\vec{r}_1 = \vec{0}$; at $s = 1$, $\vec{r}_2 = \vec{0}$.

Question 9

Construct a vector function for a helix that lies on the cylinder $(x - 2)^2 + (y + 1)^2 = 9$, begins at $(5, -1, 0)$ when $t = 0$, rises 4 units during each counterclockwise revolution, and makes two revolutions.

Question 9 – Solution

Strategy Use the cylinder center and radius for the circular components, then choose a linear height matching the pitch.



Construction Counterclockwise motion from the rightmost point is

$$x = 2 + 3 \cos t, \quad y = -1 + 3 \sin t.$$

One revolution is $\Delta t = 2\pi$ and should raise z by 4, so $z = (2/\pi)t$. Two revolutions require $0 \leq t \leq 4\pi$. Thus

$$\boxed{\vec{r}(t) = \langle 2 + 3 \cos t, -1 + 3 \sin t, 2t/\pi \rangle}, \quad 0 \leq t \leq 4\pi.$$

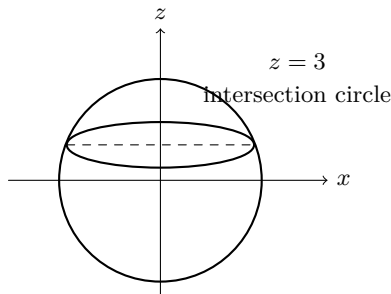
Verification The cylinder equation is identically 9, and the final height is 8.

Question 10

Find a parametrization of the curve where the sphere $x^2 + y^2 + z^2 = 25$ meets the plane $z = 3$. Give both orientations and explain why the parametrization covers the entire intersection exactly once on your chosen interval.

Question 10 – Solution

Strategy A horizontal plane cuts the sphere in a circle; determine its radius.



Circle Substituting $z = 3$ gives $x^2 + y^2 = 16$, a circle of radius 4 centered at $(0, 0, 3)$. A counterclockwise parametrization viewed from positive z is

$$\vec{r}(t) = \langle 4 \cos t, 4 \sin t, 3 \rangle, \quad 0 \leq t < 2\pi.$$

The opposite orientation is $\vec{q}(t) = \langle 4 \cos t, -4 \sin t, 3 \rangle$.

Coverage Sine and cosine generate every radius-4 circle point; the half-open interval prevents duplication of the initial point at 2π .