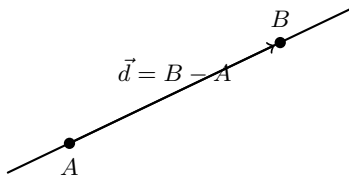


Question 1

Find vector, parametric, and symmetric equations of the line through $A = (-2, 3, 1)$ and $B = (4, -1, 7)$. Determine the point on the line whose x -coordinate is 7.

Question 1 – Solution

Strategy Use $B - A$ as a direction vector and keep one common parameter in all coordinates.



Equations A direction is $\langle 6, -4, 6 \rangle = 2 \langle 3, -2, 3 \rangle$. Thus

$$\vec{r} = \langle -2, 3, 1 \rangle + t \langle 3, -2, 3 \rangle, \quad x = -2 + 3t, \quad y = 3 - 2t, \quad z = 1 + 3t,$$

and $\boxed{(x + 2)/3 = (y - 3)/(-2) = (z - 1)/3}$.

Requested point From $-2 + 3t = 7$, $t = 3$, giving $\boxed{(7, -3, 10)}$.

Verification $t = 0$ gives A and $t = 2$ gives B .

Question 2

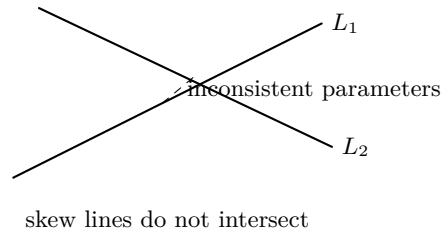
Determine whether the lines

$$L_1 : \vec{r} = \langle 1, 0, 2 \rangle + s \langle 2, -1, 3 \rangle, \quad L_2 : \vec{r} = \langle 3, -1, 1 \rangle + t \langle 1, 1, 2 \rangle$$

intersect, are parallel, or are skew. Justify the classification.

Question 2 – Solution

Strategy Equate coordinates. An intersection requires one pair (s, t) satisfying all three equations.



Coordinate equations The first two equations are $1 + 2s = 3 + t$ and $-s = -1 + t$. The second gives $t = 1 - s$; substitution into the first gives $s = 1$, hence $t = 0$.

Consistency check The third coordinates would be $2 + 3(1) = 5$ and $1 + 2(0) = 1$, so they disagree. The directions are not scalar multiples. Therefore the lines are skew.

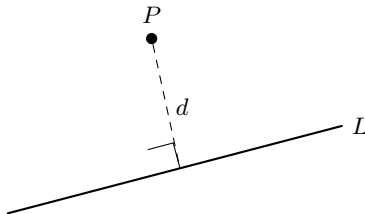
Key idea Solving only two coordinate equations can produce a false intersection; the remaining coordinate is essential.

Question 3

Find the distance from $P = (3, 1, -2)$ to the line $L : \vec{r} = \langle 1, -1, 2 \rangle + t \langle 2, 1, -2 \rangle$ and find the closest point on L .

Question 3 – Solution

Strategy At the closest point Q , the connector $P - Q$ is perpendicular to the line's direction.



Perpendicular condition Write $Q = (1 + 2t, -1 + t, 2 - 2t)$. Then

$$(P - Q) \cdot \langle 2, 1, -2 \rangle = 0.$$

Since $P - Q = \langle 2 - 2t, 2 - t, -4 + 2t \rangle$, the equation is $14 - 9t = 0$, so $t = 14/9$.

Closest point and distance Thus $Q = \boxed{(37/9, 5/9, -10/9)}$. Also $P - Q = \frac{1}{9} \langle -10, 4, -8 \rangle$, so

$$\boxed{d(P, L) = \frac{\sqrt{180}}{9} = \frac{2\sqrt{5}}{3}}.$$

Verification The dot product of $\langle -10, 4, -8 \rangle$ with $\langle 2, 1, -2 \rangle$ is zero.

Question 4

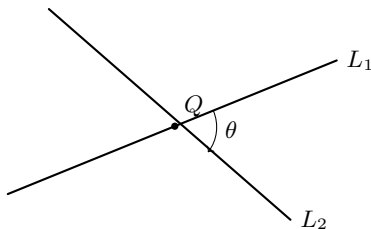
The lines

$$L_1 : \vec{r} = \langle 1, 2, -1 \rangle + s \langle 2, -1, 2 \rangle, \quad L_2 : \vec{r} = \langle 5, 0, 3 \rangle + t \langle 1, 2, -2 \rangle$$

intersect. Find their intersection and the acute angle between them.

Question 4 – Solution

Strategy Solve the coordinate equations for the intersection; use direction vectors for the angle.



Intersection Equating coordinates gives $1 + 2s = 5 + t$, $2 - s = 2t$, and $-1 + 2s = 3 - 2t$. The solution is $s = 2, t = 0$, so $Q = (5, 0, 3)$.

Angle With $\vec{d}_1 = \langle 2, -1, 2 \rangle$ and $\vec{d}_2 = \langle 1, 2, -2 \rangle$,

$$|\vec{d}_1 \cdot \vec{d}_2| = 4, \quad \|\vec{d}_1\| = 3, \quad \|\vec{d}_2\| = 3.$$

Hence $\theta = \cos^{-1}(4/9) \approx 63.6^\circ$.

Verification Substituting $s = 2$ and $t = 0$ gives the same point on both lines.

Question 5

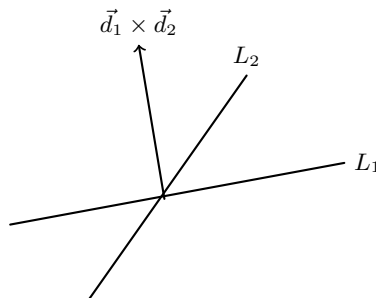
Find the line through $P = (2, -1, 4)$ whose direction is perpendicular to the directions of both lines

$$L_1 : \vec{r} = \langle 0, 1, 2 \rangle + s \langle 1, 2, -1 \rangle, \quad L_2 : \vec{r} = \langle 3, 0, -1 \rangle + t \langle 2, -1, 1 \rangle.$$

Explain why exactly one such direction exists.

Question 5 – Solution

Strategy A vector perpendicular to both directions is their cross product.



Direction Compute

$$\langle 1, 2, -1 \rangle \times \langle 2, -1, 1 \rangle = \langle 1, -3, -5 \rangle.$$

Therefore the requested line is

$$\boxed{\vec{r} = \langle 2, -1, 4 \rangle + u \langle 1, -3, -5 \rangle}.$$

Verification The new direction has dot product 0 with each original direction.

Uniqueness The original directions are not parallel, so their common perpendicular directions form the one-dimensional span of their nonzero cross product; reversing direction describes the same line.

Question 6

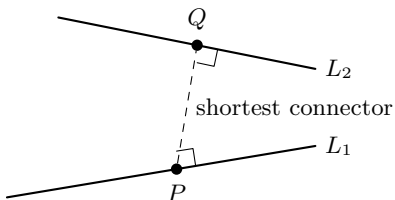
Find the shortest distance between the skew lines

$$L_1 : \vec{r} = \langle 0, 0, 0 \rangle + s \langle 1, 1, 0 \rangle, \quad L_2 : \vec{r} = \langle 0, 1, 1 \rangle + t \langle 1, -1, 0 \rangle,$$

and identify the pair of closest points.

Question 6 – Solution

Strategy The connector between closest points must be perpendicular to both line directions.



Closest points Let $P = (s, s, 0)$ and $Q = (t, 1 - t, 1)$. Requiring $(Q - P) \cdot \langle 1, 1, 0 \rangle = 0$ gives $1 - 2s = 0$, so $s = 1/2$. Requiring $(Q - P) \cdot \langle 1, -1, 0 \rangle = 0$ gives $2t - 1 = 0$, so $t = 1/2$.

Distance Thus $P = (1/2, 1/2, 0)$ and $Q = (1/2, 1/2, 1)$, with

$$\boxed{d(L_1, L_2) = |PQ| = 1}.$$

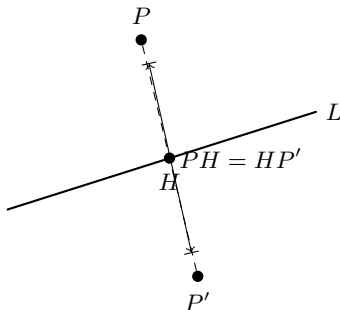
Verification $Q - P = \langle 0, 0, 1 \rangle$ is perpendicular to both direction vectors.

Question 7

Reflect the point $P = (4, 0, 1)$ across the line $L : \vec{r} = \langle 1, 1, 1 \rangle + t \langle 1, -1, 0 \rangle$. That is, find P' such that L perpendicularly bisects $\overline{PP'}$.

Question 7 – Solution

Strategy First find the perpendicular foot H on the line; then use $H = (P + P')/2$.



Projection point Write $H = (1 + t, 1 - t, 1)$. The condition $(P - H) \cdot \langle 1, -1, 0 \rangle = 0$ gives

$$\langle 3 - t, t - 1, 0 \rangle \cdot \langle 1, -1, 0 \rangle = 4 - 2t = 0,$$

so $t = 2$ and $H = (3, -1, 1)$.

Reflection Therefore $P' = 2H - P = \boxed{(2, -2, 1)}$.

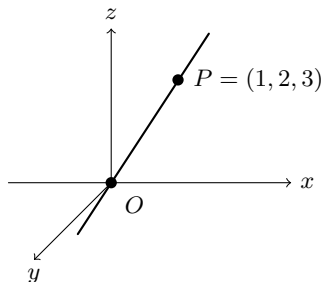
Verification The midpoint of P and P' is H , and $P - H = \langle 1, 1, 0 \rangle$ is perpendicular to the direction of L .

Question 8

Find the line that passes through $P = (1, 2, 3)$ and meets both the x -axis and the y -axis. Determine the two intercept points, or prove that no such line exists.

Question 8 – Solution

Strategy Distinct axis intercepts determine a line in the xy -plane, but both intercepts may be the origin.



Distinct intercepts A line through distinct points $X = (a, 0, 0)$ and $Y = (0, b, 0)$ lies in $z = 0$, so it cannot contain P .

Coincident intercepts The axes share the origin. The line through the origin and P is

$$\vec{r}(t) = t \langle 1, 2, 3 \rangle, \quad t \in \mathbb{R}.$$

It passes through P at $t = 1$ and meets each axis at $t = 0$. Thus both intercepts are $(0, 0, 0)$.

Key idea The distinct-point argument does not rule out coincident intercepts. This is the unique line satisfying the question.

Question 9

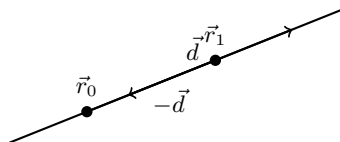
Show that

$$L_1 : \vec{r} = \langle 2, -1, 3 \rangle + t \langle 4, 2, -2 \rangle \quad \text{and} \quad L_2 : \vec{r} = \langle 8, 2, 0 \rangle + s \langle -2, -1, 1 \rangle$$

describe the same geometric line. Find the parameter conversion between t and s .

Question 9 – Solution

Strategy Check parallel directions and show that one base point lies on the other line.



different parameters, same point set

Same point set The second direction is $-\frac{1}{2}\langle 4, 2, -2 \rangle$. Also $\langle 8, 2, 0 \rangle = \langle 2, -1, 3 \rangle + \frac{3}{2}\langle 4, 2, -2 \rangle$, so the second base point lies on L_1 . Thus the lines coincide.

Parameter conversion Equating the representations gives

$$\vec{r} = \vec{r}_0 + t\vec{d} = \vec{r}_0 + \frac{3}{2}\vec{d} - \frac{s}{2}\vec{d},$$

so $\boxed{t = \frac{3}{2} - \frac{s}{2}}$, equivalently $s = 3 - 2t$.

Verification The conversion makes all three coordinates identical.

Question 10

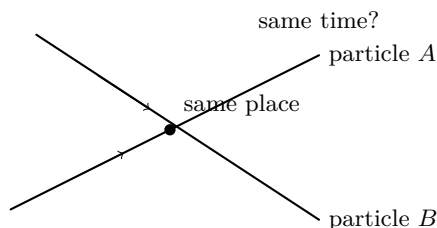
Two particles move according to

$$\vec{r}_A(t) = \langle 1, 0, 2 \rangle + t \langle 2, 1, -1 \rangle, \quad \vec{r}_B(t) = \langle 5, 2, 0 \rangle + t \langle -1, 0, 1 \rangle, \quad t \geq 0.$$

Do their paths intersect? Do the particles collide? Find their minimum separation for $t \geq 0$.

Question 10 – Solution

Strategy Path intersection permits different parameters; collision requires the same time. Minimum separation comes from minimizing the squared relative distance.



Paths Solving $\vec{r}_A(s) = \vec{r}_B(u)$ gives $s = 2, u = 0$, so the paths intersect at $(5, 2, 0)$. The particles do not collide there because they reach it at different times. Directly equating positions at the same t is inconsistent.

Minimum separation The relative position is

$$\vec{r}_A(t) - \vec{r}_B(t) = \langle -4 + 3t, -2 + t, 2 - 2t \rangle.$$

Thus $D^2(t) = 14t^2 - 36t + 24$, minimized at $t = 36/28 = 9/7$. The minimum squared distance is $6/7$, so

$$D_{\min} = \sqrt{6/7} = \frac{\sqrt{42}}{7}.$$

Verification The minimizing time is nonnegative, and the quadratic has positive leading coefficient.