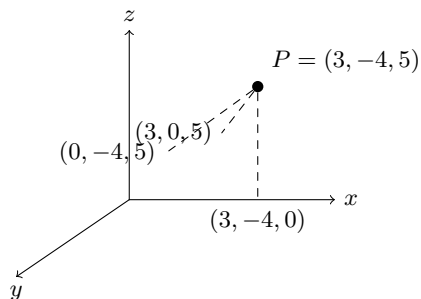


Question 1

A point is $P = (3, -4, 5)$. Find its perpendicular distances to each coordinate plane and to each coordinate axis. Identify the closest plane and the closest axis.

Question 1 – Solution

Strategy A coordinate plane is reached by changing one coordinate to zero. For an axis, two coordinates must become zero, so the distance comes from those two perpendicular components.



Coordinate planes The distances to the yz -, xz -, and xy -planes are $|3| = 3$, $|-4| = 4$, and $|5| = 5$, respectively. Thus the closest plane is the yz -plane.

Coordinate axes For the x -axis, $d = \sqrt{(-4)^2 + 5^2} = \sqrt{41}$. Similarly,

$$d(P, y\text{-axis}) = \sqrt{3^2 + 5^2} = \sqrt{34}, \quad d(P, z\text{-axis}) = \sqrt{3^2 + (-4)^2} = 5.$$

Hence the closest axis is the z -axis, at distance $\boxed{5}$.

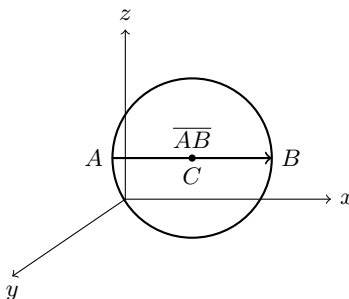
Verification Each plane distance uses exactly the omitted coordinate; each axis distance uses exactly the other two coordinates.

Question 2

The endpoints of a diameter of a sphere are $A = (-2, 1, 4)$ and $B = (6, -3, 0)$. Find the sphere's center, radius, and equation. Determine where the sphere intersects the z -axis.

Question 2 – Solution

Strategy The center is the midpoint of the diameter, and the radius is half its length. Points on the z -axis have the form $(0, 0, z)$.



Center and radius The midpoint is $C = (2, -1, 2)$. Also,

$$|AB| = \sqrt{8^2 + (-4)^2 + (-4)^2} = 4\sqrt{6}, \quad r = 2\sqrt{6}.$$

Therefore the sphere is

$$(x - 2)^2 + (y + 1)^2 + (z - 2)^2 = 24.$$

z -axis intersections Set $x = y = 0$: $4 + 1 + (z - 2)^2 = 24$, so $(z - 2)^2 = 19$. The intersection points are $(0, 0, 2 \pm \sqrt{19})$.

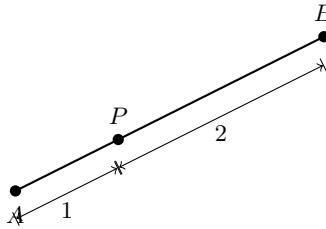
Verification Both A and B are $2\sqrt{6}$ units from C , and their midpoint is C .

Question 3

Point P divides the segment from $A = (-5, 2, 7)$ to $B = (4, -4, 1)$ internally in the ratio $AP : PB = 1 : 2$. Find P , then verify the ratio using distances.

Question 3 – Solution

Strategy Moving one-third of the way from A to B produces the required $1 : 2$ division.



Section formula Since $B - A = \langle 9, -6, -6 \rangle$,

$$P = A + \frac{1}{3}(B - A) = (-5, 2, 7) + (3, -2, -2) = \boxed{(-2, 0, 5)}.$$

Distance check We have $P - A = \langle 3, -2, -2 \rangle$ and $B - P = \langle 6, -4, -4 \rangle = 2(P - A)$. Therefore $AP = \sqrt{17}$ and $PB = 2\sqrt{17}$, giving $AP : PB = 1 : 2$.

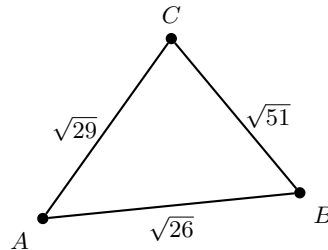
Key idea An internal ratio determines an affine weighted average; the distance check also confirms that P lies between the endpoints.

Question 4

The points $A = (1, 0, 2)$, $B = (4, 4, 1)$, and $C = (-1, 3, 6)$ form a triangle. Classify it as scalene, isosceles, or equilateral, and determine whether it is acute, right, or obtuse.

Question 4 – Solution

Strategy Compare squared side lengths; this avoids unnecessary square roots. The converse of the Pythagorean theorem classifies the largest angle.



Side lengths Direct calculation gives

$$AB^2 = 3^2 + 4^2 + (-1)^2 = 26, \quad AC^2 = (-2)^2 + 3^2 + 4^2 = 29, \quad BC^2 = (-5)^2 + (-1)^2 + 5^2 = 51.$$

The three lengths differ, so the triangle is scalene.

Angle classification The longest side is BC . Since $BC^2 = 51 < 26 + 29 = 55$, the angle opposite BC is acute. The other angles are necessarily smaller, so the triangle is scalene and acute.

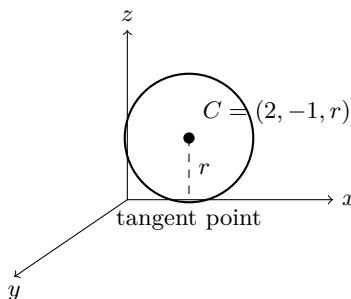
Verification The triangle inequality holds; for example, $\sqrt{26} + \sqrt{29} > \sqrt{51}$.

Question 5

A sphere has center $C = (2, -1, c)$, is tangent to the xy -plane, lies entirely above that plane, and passes through $P = (5, 3, 2)$. Find c and the sphere's equation.

Question 5 – Solution

Strategy Tangency to the xy -plane makes the radius equal to the center's height. The condition that the sphere lies above the plane forces $c > 0$.



Build the equation Here $r = c$. Because P is on the sphere,

$$(5 - 2)^2 + (3 + 1)^2 + (2 - c)^2 = c^2.$$

Thus $25 + (2 - c)^2 = c^2$, so $29 - 4c = 0$ and $c = 29/4$.

Result The radius is $29/4$, and

$$\boxed{(x - 2)^2 + (y + 1)^2 + \left(z - \frac{29}{4}\right)^2 = \left(\frac{29}{4}\right)^2}.$$

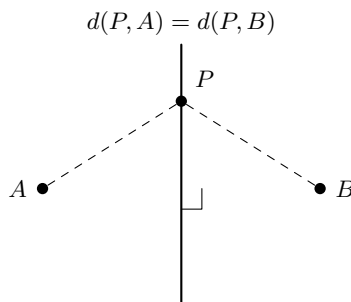
Verification The lowest z -coordinate is $c - r = 0$, so the sphere is tangent to the plane and otherwise lies above it.

Question 6

Describe geometrically the set of all points $P = (x, y, z)$ equidistant from $A = (2, -1, 3)$ and $B = (-4, 5, 1)$. Derive a Cartesian equation for the set and show that the midpoint of AB belongs to it.

Question 6 – Solution

Strategy Equate squared distances. Quadratic terms cancel, revealing the perpendicular-bisector surface.



Derivation Set

$$(x - 2)^2 + (y + 1)^2 + (z - 3)^2 = (x + 4)^2 + (y - 5)^2 + (z - 1)^2.$$

After expanding and simplifying, $\boxed{3x - 3y + z + 7 = 0}$.

Interpretation This is the plane perpendicular to $\overrightarrow{AB} = \langle -6, 6, -2 \rangle$ through the midpoint $M = (-1, 2, 2)$. Indeed, $3(-1) - 3(2) + 2 + 7 = 0$.

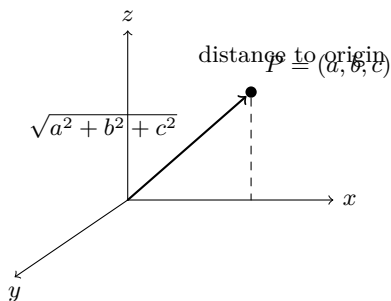
Verification Interchanging A and B does not change the locus, and every point in the plane has equal squared distance to the endpoints.

Question 7

A point $P = (a, b, c)$ is 7 units from the origin. Its distances to the yz - and xz -planes are 2 and 3, respectively, and P lies in the octant $x < 0, y > 0, z < 0$. Find P .

Question 7 – Solution

Strategy Distances to the yz - and xz -planes determine $|a|$ and $|b|$. The octant fixes signs, and the distance to the origin determines c .



Known coordinates We have $|a| = 2$ and $|b| = 3$, so $a = -2$ and $b = 3$.

Use the radial distance Since $a^2 + b^2 + c^2 = 49$,

$$4 + 9 + c^2 = 49 \implies c^2 = 36.$$

The octant condition $z < 0$ selects $c = -6$. Hence $\boxed{P = (-2, 3, -6)}$.

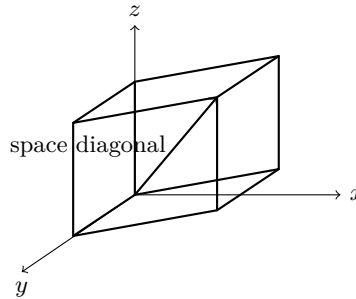
Verification Its distance from the origin is $\sqrt{4 + 9 + 36} = 7$, and its plane distances are 2 and 3 as required.

Question 8

A rectangular box has one vertex at the origin and the opposite vertex $P = (a, b, c)$ in the first octant. Its space diagonal has length 13, its xy -face diagonal has length 5, and the y -edge is longer than the x -edge. If the side lengths are positive integers, determine P .

Question 8 – Solution

Strategy Use the face diagonal to determine two integer side lengths, then use the space diagonal for the third.



Base dimensions The positive integer side lengths satisfying $a^2 + b^2 = 25$ are 3 and 4. Since the y -edge is longer than the x -edge, $a = 3$ and $b = 4$.

Third dimension The space diagonal gives

$$a^2 + b^2 + c^2 = 169 \implies 25 + c^2 = 169,$$

so $c = 12$. Therefore $\boxed{P = (3, 4, 12)}$.

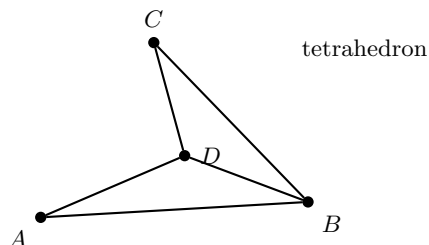
Verification The face diagonal is $\sqrt{3^2 + 4^2} = 5$ and the space diagonal is $\sqrt{3^2 + 4^2 + 12^2} = 13$.

Question 9

Four points are $A = (1, 1, 1)$, $B = (5, 1, 1)$, $C = (3, 1 + 2\sqrt{3}, 1)$, and $D = (3, 1 + 2/\sqrt{3}, 1 + 4\sqrt{2/3})$. Prove that they are the vertices of a regular tetrahedron and find its center.

Question 9 – Solution

Strategy A tetrahedron is regular if all six pairwise distances agree. Symmetry suggests that its center is the average of its four vertices.



Edge lengths Computation gives $AB = AC = BC = 4$. Also,

$$AD^2 = BD^2 = 4 + \frac{4}{3} + \frac{32}{3} = 16, \quad CD^2 = \frac{16}{3} + \frac{32}{3} = 16.$$

Thus all six edges have length 4, proving the tetrahedron is regular.

Center Averaging the vertices gives

$$G = \left(3, 1 + \frac{2\sqrt{3}}{3}, 1 + \sqrt{\frac{2}{3}} \right).$$

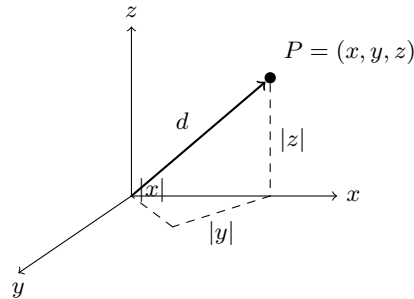
Verification By symmetry—or by direct substitution— G is equidistant from all four vertices.

Question 10

Derive the distance formula between $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$ using the Pythagorean theorem twice. Then use it to find all values of t for which the distance from $(t, 2t, -1)$ to $(1, -2, 3)$ is 6.

Question 10 – Solution

Strategy First combine the x - and y -changes into a planar diagonal; then combine that diagonal with the z -change.



Derivation The planar displacement has length $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. A second right triangle therefore gives

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

Application Set the squared distance equal to 36:

$$(t - 1)^2 + (2t + 2)^2 + (-4)^2 = 36.$$

This becomes $5t^2 + 6t - 15 = 0$, so

$$t = \frac{-3 \pm 2\sqrt{21}}{5}.$$

Verification Substitution into the quadratic recovers squared distance 36; both values are valid because distance is nonnegative.