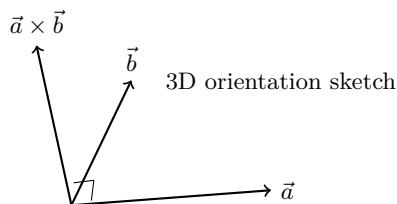


Question 1

Compute $\vec{a} \times \vec{b}$ for $\vec{a} = \langle 1, 2, -1 \rangle$ and $\vec{b} = \langle 3, 0, 4 \rangle$, then verify it is perpendicular to both vectors.

Question 1 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 Expand the determinant: $\vec{a} \times \vec{b} = \langle 2(4) - (-1)(0), (-1)(3) - 1(4), 1(0) - 2(3) \rangle = \langle 8, -7, -6 \rangle$.

Step 2 Check $(\vec{a} \times \vec{b}) \cdot \vec{a} = 8 - 14 + 6 = 0$.

Step 3 Also $(\vec{a} \times \vec{b}) \cdot \vec{b} = 24 - 24 = 0$, so $\boxed{\vec{a} \times \vec{b} = \langle 8, -7, -6 \rangle}$ is perpendicular to both.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

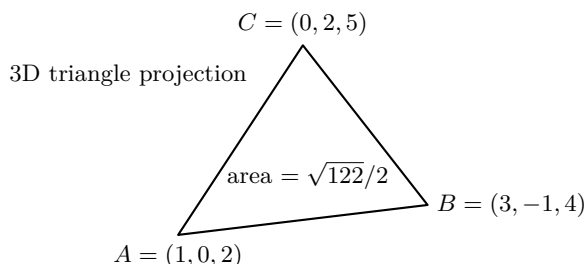
Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.

Question 2

Find the area of the triangle with vertices $A = (1, 0, 2)$, $B = (3, -1, 4)$, and $C = (0, 2, 5)$.

Question 2 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 Form two sides: $\overrightarrow{AB} = \langle 2, -1, 2 \rangle$ and $\overrightarrow{AC} = \langle -1, 2, 3 \rangle$.

Step 2 Their cross product is $\langle -7, -8, 3 \rangle$, whose magnitude is $\sqrt{49 + 64 + 9} = \sqrt{122}$.

Step 3 A triangle has half the parallelogram area, so $A = \sqrt{122}/2$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

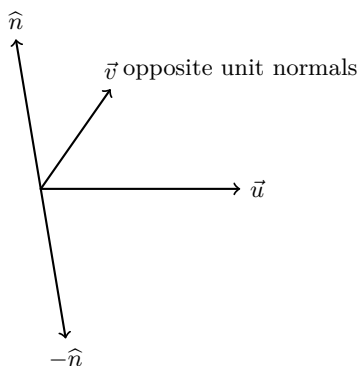
Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.

Question 3

Find both unit vectors perpendicular to $\langle 2, 1, 0 \rangle$ and $\langle 1, -1, 3 \rangle$.

Question 3 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 Take a cross product: $\langle 2, 1, 0 \rangle \times \langle 1, -1, 3 \rangle = \langle 3, -6, -3 \rangle = 3\langle 1, -2, -1 \rangle$.

Step 2 Its magnitude is $3\sqrt{6}$.

Step 3 Normalize and include both orientations: $\pm \frac{1}{\sqrt{6}} \langle 1, -2, -1 \rangle$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

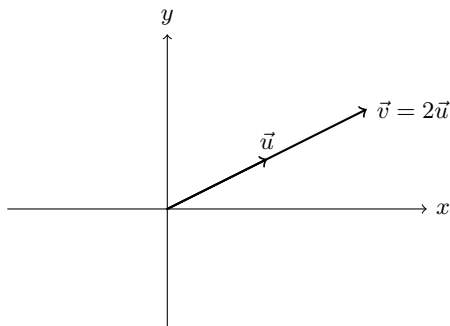
Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.

Question 4

For what values of k are $\vec{u} = \langle 1, k, 2 \rangle$ and $\vec{v} = \langle 2, 2k, 4 \rangle$ parallel? What is their cross product then?

Question 4 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 Observe directly that $\vec{v} = 2\vec{u}$ for every real k .

Step 2 Therefore the vectors are parallel for $\boxed{\text{all } k \in \mathbb{R}}$.

Step 3 The cross product of parallel vectors is the zero vector, so $\boxed{\vec{u} \times \vec{v} = \vec{0}}$ for every k .

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

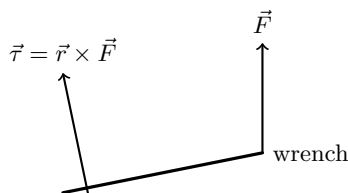
Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.

Question 5

A wrench extends from the origin along $\vec{r} = \langle 0.30, 0.10, 0 \rangle$ meters. A force $\vec{F} = \langle 0, 40, -20 \rangle$ newtons is applied. Find the torque vector and its magnitude.

Question 5 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 Torque is $\vec{\tau} = \vec{r} \times \vec{F}$.

Step 2 Compute $\vec{\tau} = \langle 0.10(-20), -0.30(-20), 0.30(40) \rangle = \langle -2, 6, 12 \rangle \text{ N}\cdot\text{m}$.

Step 3 Its magnitude is $\sqrt{4 + 36 + 144} = \boxed{2\sqrt{46} \text{ N}\cdot\text{m}}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

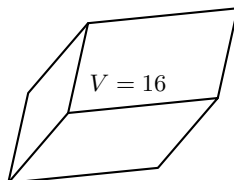
Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.

Question 6

Find the volume of the parallelepiped determined by $\vec{a} = \langle 1, 2, 0 \rangle$, $\vec{b} = \langle 0, 1, 3 \rangle$, and $\vec{c} = \langle 2, -1, 1 \rangle$.

Question 6 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 Volume is the absolute scalar triple product $|\vec{a} \cdot (\vec{b} \times \vec{c})|$.

Step 2 Compute $\vec{b} \times \vec{c} = \langle 4, 6, -2 \rangle$.

Step 3 Then $\vec{a} \cdot (\vec{b} \times \vec{c}) = 4 + 12 = 16$, so $\boxed{V = 16}$ cubic units.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

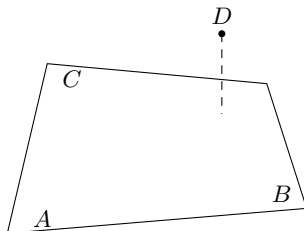
Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.

Question 7

Determine whether the points $A = (1, 2, 0)$, $B = (3, 1, 4)$, $C = (-1, 5, 1)$, and $D = (5, -2, 7)$ are coplanar.

Question 7 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 Use vectors from A : $\overrightarrow{AB} = \langle 2, -1, 4 \rangle$, $\overrightarrow{AC} = \langle -2, 3, 1 \rangle$, $\overrightarrow{AD} = \langle 4, -4, 7 \rangle$.

Step 2 Compute $\overrightarrow{AB} \times \overrightarrow{AC} = \langle -13, -10, 4 \rangle$.

Step 3 Dot with $\overrightarrow{AD}$: $-52 + 40 + 28 = 16 \neq 0$. Therefore the points are not coplanar.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

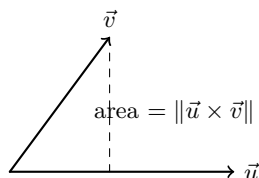
Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.

Question 8

Prove the area identity $\|\vec{u} \times \vec{v}\|^2 = \|\vec{u}\|^2\|\vec{v}\|^2 - (\vec{u} \cdot \vec{v})^2$ using the angle between the vectors.

Question 8 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 If either vector is zero, both sides are zero. Otherwise, let θ be the angle between $\vec{u}$ and $\vec{v}$. Then $\|\vec{u} \times \vec{v}\| = \|\vec{u}\|\|\vec{v}\|\sin\theta$.

Step 2 Squaring and using $\sin^2\theta = 1 - \cos^2\theta$ gives $\|\vec{u}\|^2\|\vec{v}\|^2(1 - \cos^2\theta)$.

Step 3 Since $\vec{u} \cdot \vec{v} = \|\vec{u}\|\|\vec{v}\|\cos\theta$, substitution yields the claimed identity. Proved

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

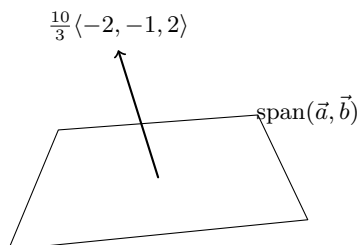
Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.

Question 9

Find a vector of magnitude 10 perpendicular to the plane spanned by $\vec{a} = \langle 1, 0, 1 \rangle$ and $\vec{b} = \langle 0, 2, 1 \rangle$, with positive z -component.

Question 9 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 A normal direction is $\vec{a} \times \vec{b} = \langle -2, -1, 2 \rangle$.

Step 2 Its magnitude is 3, and its z -component is positive.

Step 3 Scale it to length 10: $\vec{n} = \frac{10}{3}\langle -2, -1, 2 \rangle$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

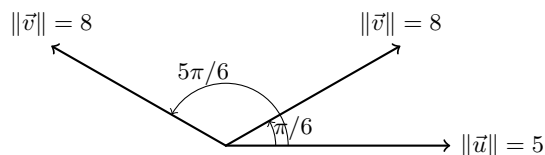
Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.

Question 10

Vectors $\vec{u}$ and $\vec{v}$ have lengths 5 and 8, and $\|\vec{u} \times \vec{v}\| = 20$. Find every possible angle between them in $[0, \pi]$.

Question 10 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 Use $\|\vec{u} \times \vec{v}\| = \|\vec{u}\|\|\vec{v}\|\sin\theta$: $20 = 40\sin\theta$.

Step 2 Thus $\sin\theta = 1/2$.

Step 3 On $[0, \pi]$, the two solutions are $\theta = \pi/6$ or $5\pi/6$. The cross-product magnitude cannot distinguish acute from obtuse orientation.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.