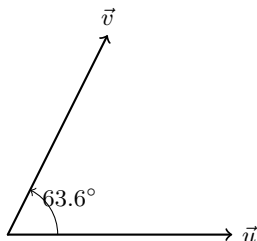


Question 1

Find the angle between $\vec{u} = \langle 1, 2, 2 \rangle$ and $\vec{v} = \langle 2, -1, 2 \rangle$.

Question 1 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 Compute $\vec{u} \cdot \vec{v} = 2 - 2 + 4 = 4$.

Step 2 Both magnitudes are 3, so $\cos \theta = 4/(3 \cdot 3) = 4/9$.

Step 3 Therefore $\theta = \cos^{-1}(4/9) \approx 63.6^\circ$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

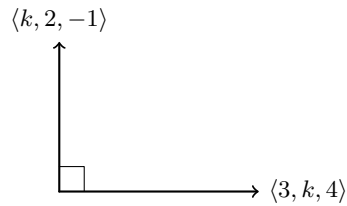
Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.

Question 2

Find all k such that $\langle k, 2, -1 \rangle$ is perpendicular to $\langle 3, k, 4 \rangle$.

Question 2 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 Perpendicular vectors have zero dot product.

Step 2 Set $3k + 2k - 4 = 0$, which simplifies to $5k = 4$.

Step 3 Thus $k = 4/5$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

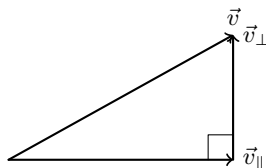
Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.

Question 3

Decompose $\vec{v} = \langle 5, 1, -2 \rangle$ into components parallel and perpendicular to $\vec{u} = \langle 1, 2, 2 \rangle$.

Question 3 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 The parallel component is $\text{proj}_{\vec{u}} \vec{v} = \frac{\vec{v} \cdot \vec{u}}{\vec{u} \cdot \vec{u}} \vec{u}$. Here $\vec{v} \cdot \vec{u} = 3$ and $\vec{u} \cdot \vec{u} = 9$.

Step 2 So $\vec{v}_{\parallel} = \frac{1}{3} \langle 1, 2, 2 \rangle = \langle 1/3, 2/3, 2/3 \rangle$.

Step 3 Subtract: $\vec{v}_{\perp} = \vec{v} - \vec{v}_{\parallel} = \boxed{\langle 14/3, 1/3, -8/3 \rangle}$. Their dot product is zero.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

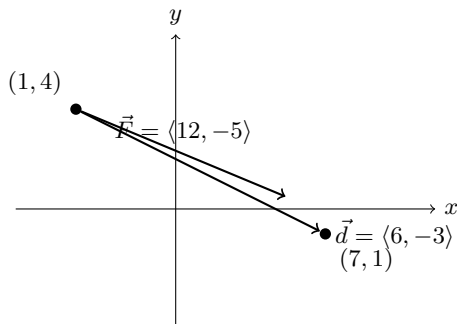
Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.

Question 4

A force $\vec{F} = \langle 12, -5 \rangle$ moves an object from $(1, 4)$ to $(7, 1)$. Find the work done.

Question 4 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 The displacement is $\vec{d} = \langle 7 - 1, 1 - 4 \rangle = \langle 6, -3 \rangle$.

Step 2 Work is the dot product: $W = \vec{F} \cdot \vec{d}$.

Step 3 Thus $W = 12(6) + (-5)(-3) = 72 + 15 = \boxed{87}$ force-distance units.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

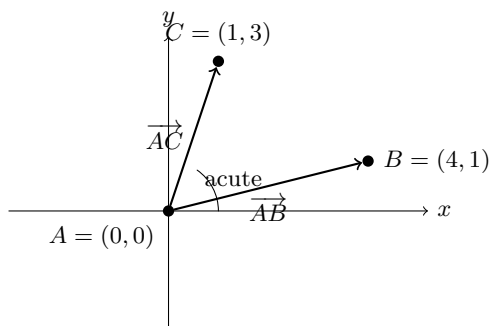
Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.

Question 5

Determine whether the triangle with vertices $A = (0, 0)$, $B = (4, 1)$, and $C = (1, 3)$ has an acute, right, or obtuse angle at A .

Question 5 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 The sides from A are $\vec{AB} = \langle 4, 1 \rangle$ and $\vec{AC} = \langle 1, 3 \rangle$.

Step 2 Their dot product is $4(1) + 1(3) = 7 > 0$.

Step 3 A positive dot product means the included angle has positive cosine, so the angle at A is acute.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

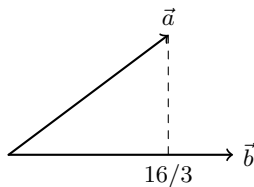
Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.

Question 6

Find the scalar projection of $\vec{a} = \langle -2, 5, 4 \rangle$ onto $\vec{b} = \langle 1, 2, 2 \rangle$, and explain the sign.

Question 6 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 Compute $\vec{a} \cdot \vec{b} = -2 + 10 + 8 = 16$ and $\|\vec{b}\| = 3$.

Step 2 The scalar projection is $\text{comp}_{\vec{b}} \vec{a} = (\vec{a} \cdot \vec{b}) / \|\vec{b}\| = \boxed{16/3}$.

Step 3 It is positive because $\vec{a}$ has a net component in the same direction as $\vec{b}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

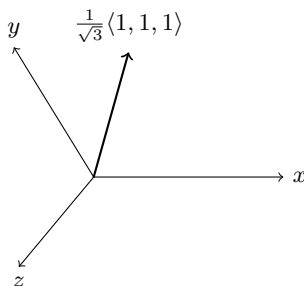
Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.

Question 7

A unit vector $\vec{n}$ makes equal acute angles with the positive coordinate axes in $\mathbb{R}^3$. Find $\vec{n}$ and the common angle.

Question 7 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 Equal direction cosines imply $\vec{n} = \langle c, c, c \rangle$ with $c > 0$.

Step 2 The unit condition gives $3c^2 = 1$, so $c = 1/\sqrt{3}$.

Step 3 Hence $\vec{n} = \frac{1}{\sqrt{3}}\langle 1, 1, 1 \rangle$ and $\theta = \cos^{-1}(1/\sqrt{3})$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

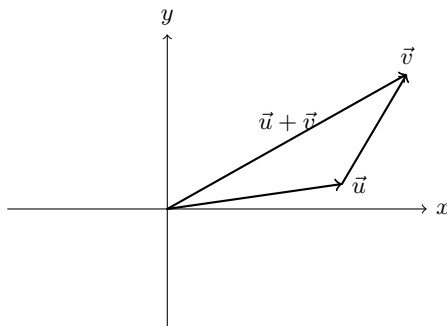
Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.

Question 8

Prove the identity $\|\vec{u} + \vec{v}\|^2 = \|\vec{u}\|^2 + 2\vec{u} \cdot \vec{v} + \|\vec{v}\|^2$.

Question 8 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 Start with the squared norm: $\|\vec{u} + \vec{v}\|^2 = (\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v})$.

Step 2 Distribute the dot product to obtain $\vec{u} \cdot \vec{u} + \vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{u} + \vec{v} \cdot \vec{v}$.

Step 3 Use symmetry and $\vec{u} \cdot \vec{u} = \|\vec{u}\|^2$ to get the stated identity. Proved

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

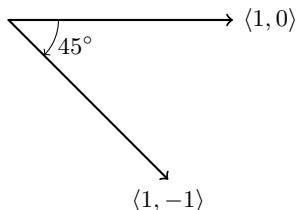
Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.

Question 9

For which real t is the angle between $\langle 1, t \rangle$ and $\langle 1, -1 \rangle$ equal to 45° ?

Question 9 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 Use the cosine formula: $(1 - t)/(\sqrt{1 + t^2}\sqrt{2}) = \sqrt{2}/2$.

Step 2 This simplifies to $1 - t = \sqrt{1 + t^2}$. The right side is nonnegative, so $t \leq 1$. Squaring gives $1 - 2t + t^2 = 1 + t^2$, hence $t = 0$.

Step 3 It satisfies the original equation, so $\boxed{t = 0}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

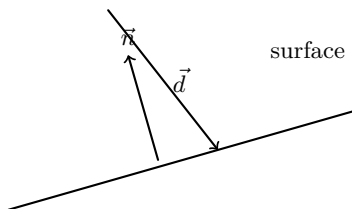
Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.

Question 10

A spotlight beam points along $\vec{d} = \langle 2, -1, 2 \rangle$. A surface has unit normal $\vec{n} = \frac{1}{3}\langle 1, 2, 2 \rangle$. Find the magnitude of the component of $\vec{d}$ normal to the surface.

Question 10 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 Because $\vec{n}$ is a unit vector, the normal component magnitude is $|\vec{d} \cdot \vec{n}|$.

Step 2 Compute $\vec{d} \cdot \vec{n} = (2 - 2 + 4)/3 = 4/3$.

Step 3 Therefore the requested magnitude is $\boxed{4/3}$. The absolute value makes the result independent of the chosen normal orientation.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.