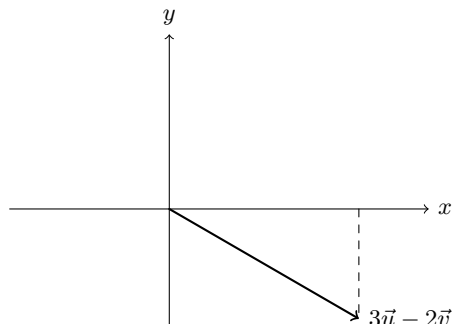


Question 1

Given $\vec{u} = \langle 2, -1, 3 \rangle$ and $\vec{v} = \langle -4, 5, 1 \rangle$, find $3\vec{u} - 2\vec{v}$ and its magnitude.

Question 1 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.



Step 1 Scale componentwise: $3\vec{u} = \langle 6, -3, 9 \rangle$ and $2\vec{v} = \langle -8, 10, 2 \rangle$.

Step 2 Subtract to get $3\vec{u} - 2\vec{v} = \langle 14, -13, 7 \rangle$.

Step 3 Its magnitude is $\sqrt{196 + 169 + 49} = \boxed{\sqrt{414} = 3\sqrt{46}}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

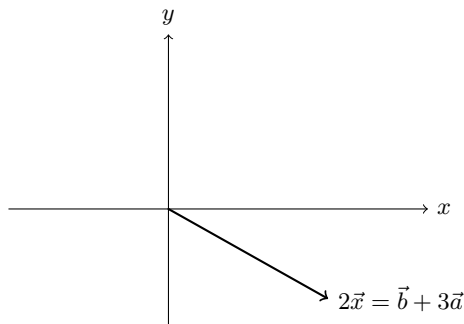
Key idea Vector equations behave like ordinary algebra provided that every operation is performed component by component. A final component check is often the fastest verification.

Question 2

Solve the vector equation $2\vec{x} - 3\vec{a} = \vec{b}$ when $\vec{a} = \langle 1, -2, 4 \rangle$ and $\vec{b} = \langle 5, 0, -6 \rangle$.

Question 2 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.



Step 1 Isolate the unknown: $2\vec{x} = \vec{b} + 3\vec{a}$.

Step 2 Compute $\vec{b} + 3\vec{a} = \langle 5, 0, -6 \rangle + \langle 3, -6, 12 \rangle = \langle 8, -6, 6 \rangle$.

Step 3 Divide by 2: $\vec{x} = \langle 4, -3, 3 \rangle$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

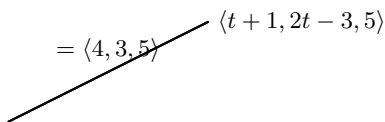
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Question 3

For what value of t are $\langle t + 1, 2t - 3, 5 \rangle$ and $\langle 4, 3, 5 \rangle$ equal? If none exists, identify the obstruction.

Question 3 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.


$$= \langle 4, 3, 5 \rangle \quad \langle t + 1, 2t - 3, 5 \rangle$$

Step 1 Equal vectors require equality of every component.

Step 2 The first component gives $t + 1 = 4$, so $t = 3$; the second gives $2t - 3 = 3$, also $t = 3$.

Step 3 The third components already agree, so the unique value is $\boxed{t = 3}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

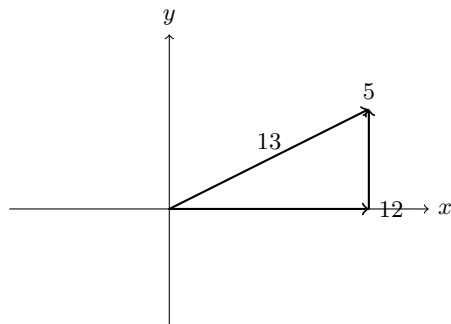
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Question 4

A boat moves at 12 km/h due east relative to the water while a current flows at 5 km/h due north. Find the ground velocity, speed, and direction north of east.

Question 4 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.



Step 1 Add the velocity vectors: $\vec{v}_g = \langle 12, 0 \rangle + \langle 0, 5 \rangle = \langle 12, 5 \rangle$.

Step 2 The ground speed is $\sqrt{12^2 + 5^2} = 13$ km/h.

Step 3 The direction satisfies $\tan \theta = 5/12$, so $\theta = \tan^{-1}(5/12) \approx 22.6^\circ$ north of east.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

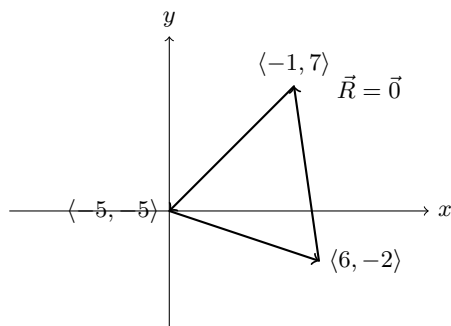
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Question 5

Three forces $\langle 6, -2 \rangle$, $\langle -1, 7 \rangle$, and $\langle p, -5 \rangle$ act on an object. Find p so the horizontal component of the resultant vanishes, then find the resultant. Is it a nonzero vertical force?

Question 5 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.



Step 1 Add components: $\vec{R} = \langle 6 - 1 + p, -2 + 7 - 5 \rangle = \langle p + 5, 0 \rangle$.

Step 2 A vertical vector must have horizontal component zero, so $p + 5 = 0$ and $p = -5$.

Step 3 Here the vertical component is also zero, so the resultant is the zero vector: $\boxed{\vec{R} = \vec{0}}$. This is equilibrium. The zero vector has no direction, so no value of p produces a nonzero vertical resultant.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

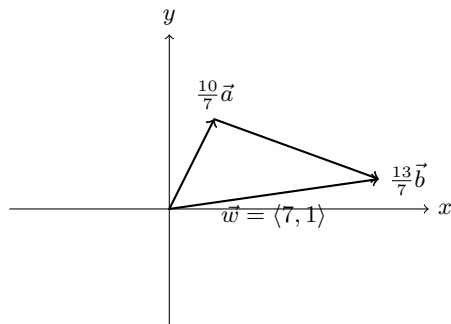
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Question 6

Write $\vec{w} = \langle 7, 1 \rangle$ as a linear combination of $\vec{a} = \langle 1, 2 \rangle$ and $\vec{b} = \langle 3, -1 \rangle$.

Question 6 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.



Step 1 Let $\vec{w} = c\vec{a} + d\vec{b}$. Components give $c + 3d = 7$ and $2c - d = 1$.

Step 2 From the second equation $d = 2c - 1$. Substitute: $c + 3(2c - 1) = 7$, so $7c = 10$.

Step 3 Thus $c = 10/7$ and $d = 13/7$, giving $\vec{w} = \frac{10}{7}\vec{a} + \frac{13}{7}\vec{b}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

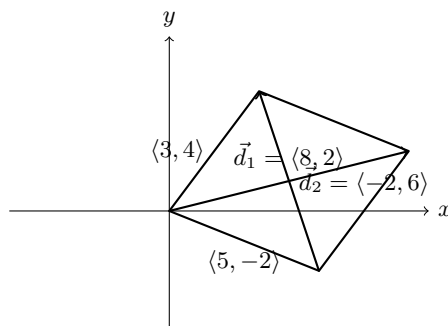
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Question 7

The diagonals of a parallelogram are $\vec{d}_1 = \langle 8, 2 \rangle$ and $\vec{d}_2 = \langle -2, 6 \rangle$. Find possible adjacent side vectors.

Question 7 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.



Step 1 If adjacent sides are $\vec{a}, \vec{b}$, then $\vec{d}_1 = \vec{a} + \vec{b}$ and $\vec{d}_2 = \vec{a} - \vec{b}$.

Step 2 Add and divide by two: $\vec{a} = (\vec{d}_1 + \vec{d}_2)/2 = \langle 3, 4 \rangle$.

Step 3 Subtract and divide by two: $\vec{b} = (\vec{d}_1 - \vec{d}_2)/2 = \boxed{\langle 5, -2 \rangle}$. Thus one ordered pair is $\boxed{\langle 3, 4 \rangle, \langle 5, -2 \rangle}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

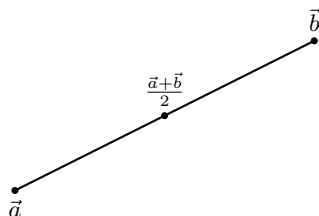
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Question 8

Prove using components that the midpoint of points with position vectors $\vec{a}$ and $\vec{b}$ has position vector $(\vec{a} + \vec{b})/2$.

Question 8 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.



Step 1 Write $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$.

Step 2 Their average is $\frac{1}{2}(\vec{a} + \vec{b}) = \left\langle \frac{a_1+b_1}{2}, \frac{a_2+b_2}{2}, \frac{a_3+b_3}{2} \right\rangle$.

Step 3 Each coordinate is the ordinary midpoint of the corresponding coordinates, so this is exactly the midpoint position vector. Proved

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

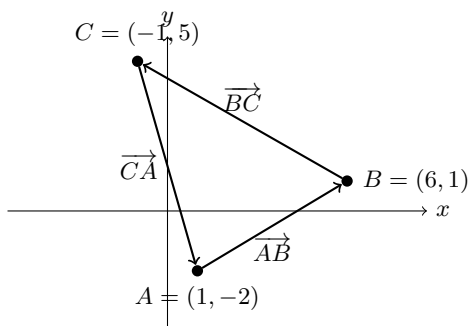
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Question 9

A triangle has vertices $A = (1, -2)$, $B = (6, 1)$, and $C = (-1, 5)$. Verify the closed-path identity $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$.

Question 9 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.



Step 1 Compute $\vec{AB} = \langle 5, 3 \rangle$, $\vec{BC} = \langle -7, 4 \rangle$, and $\vec{CA} = \langle 2, -7 \rangle$.

Step 2 Add the three: $\langle 5 - 7 + 2, 3 + 4 - 7 \rangle = \langle 0, 0 \rangle$.

Step 3 Therefore $\boxed{\vec{AB} + \vec{BC} + \vec{CA} = \vec{0}}$, as any closed displacement path must.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

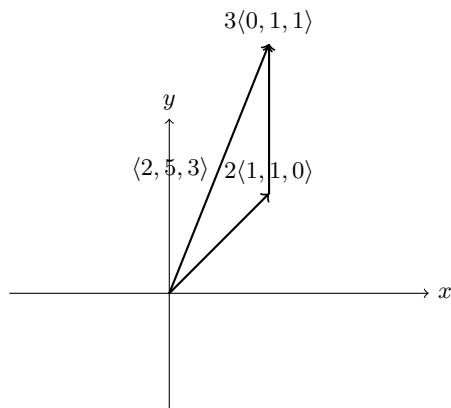
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Question 10

Find constants a, b so that $a\langle 1, 1, 0 \rangle + b\langle 0, 1, 1 \rangle = \langle 2, 5, 3 \rangle$, or prove it is impossible.

Question 10 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.



Step 1 The linear combination is $\langle a, a + b, b \rangle$.

Step 2 The first and third components force $a = 2$ and $b = 3$.

Step 3 Then $a + b = 5$, matching the middle component. Hence $\boxed{a = 2, b = 3}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

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