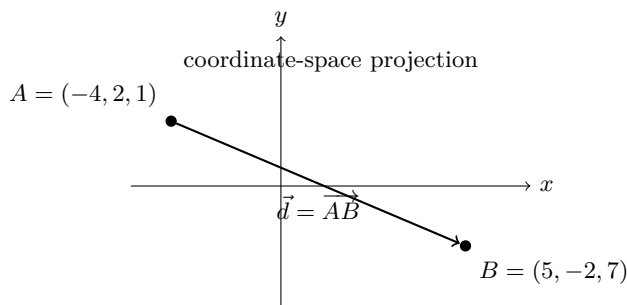


Question 1

A rescue drone travels from $A = (-4, 2, 1)$ to $B = (5, -2, 7)$. Find the displacement vector, its magnitude, and the corresponding unit direction vector.

Question 1 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 Subtract initial coordinates from terminal coordinates: $\vec{d} = B - A = \langle 9, -4, 6 \rangle$.

Step 2 Compute its length: $\|\vec{d}\| = \sqrt{9^2 + (-4)^2 + 6^2} = \sqrt{133}$.

Step 3 Normalize: $\hat{d} = \vec{d}/\|\vec{d}\| = \frac{1}{\sqrt{133}} \langle 9, -4, 6 \rangle$. The displacement magnitude is $\sqrt{133}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

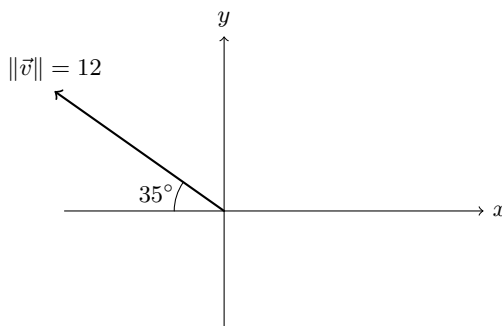
Key idea A vector separates two ideas: its components describe displacement, while its magnitude and unit vector describe size and direction. Always check signs and confirm that a unit vector has magnitude 1.

Question 2

A vector in the plane has magnitude 12, points into Quadrant II, and makes an angle of 35° with the negative x -axis. Write it in component form.

Question 2 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 Its standard-position angle is $180^\circ - 35^\circ = 145^\circ$.

Step 2 Use $\vec{v} = \|\vec{v}\|\langle \cos \theta, \sin \theta \rangle$.

Step 3 Thus $\boxed{\vec{v} = 12\langle -\cos 35^\circ, \sin 35^\circ \rangle}$. The signs agree with Quadrant II.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

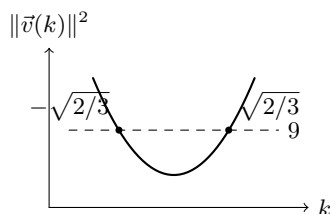
Key idea A vector separates two ideas: its components describe displacement, while its magnitude and unit vector describe size and direction. Always check signs and confirm that a unit vector has magnitude 1.

Question 3

Find all real k for which $\vec{v} = \langle k, 2k - 1, k + 2 \rangle$ has magnitude 3.

Question 3 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 Square the magnitude equation: $k^2 + (2k - 1)^2 + (k + 2)^2 = 9$.

Step 2 Expanding gives $6k^2 + 5 = 9$, so $3k^2 = 2$.

Step 3 Therefore $k = \pm\sqrt{2/3}$; substitution verifies both values.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

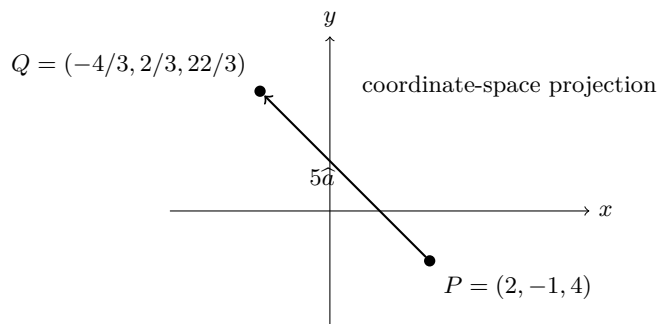
Key idea A vector separates two ideas: its components describe displacement, while its magnitude and unit vector describe size and direction. Always check signs and confirm that a unit vector has magnitude 1.

Question 4

The point $P = (2, -1, 4)$ is translated five units in the direction of $\vec{a} = \langle -2, 1, 2 \rangle$. Find the image point Q .

Question 4 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 First normalize: $\|\vec{a}\| = 3$, so $\hat{a} = \frac{1}{3}\langle -2, 1, 2 \rangle$.

Step 2 A five-unit translation has vector $5\hat{a} = \langle -10/3, 5/3, 10/3 \rangle$.

Step 3 Add it to P : $Q = P + 5\hat{a} = \left(-\frac{4}{3}, \frac{2}{3}, \frac{22}{3} \right)$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

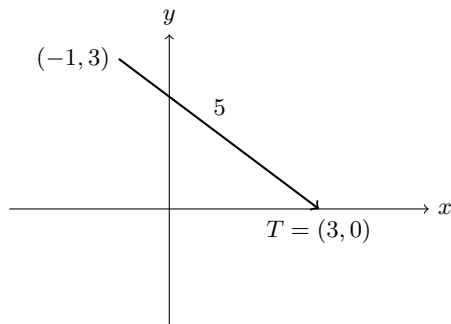
Key idea A vector separates two ideas: its components describe displacement, while its magnitude and unit vector describe size and direction. Always check signs and confirm that a unit vector has magnitude 1.

Question 5

A vector $\vec{u}$ begins at $(-1, 3)$ and ends somewhere on the positive x -axis. If $\|\vec{u}\| = 5$, find every possible terminal point.

Question 5 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 Write the terminal point as $T = (t, 0)$ with $t > 0$. Then $\vec{u} = \langle t + 1, -3 \rangle$.

Step 2 The length condition gives $(t + 1)^2 + 9 = 25$, hence $t + 1 = \pm 4$.

Step 3 The candidates are $t = 3, -5$; positivity leaves $\boxed{T = (3, 0)}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

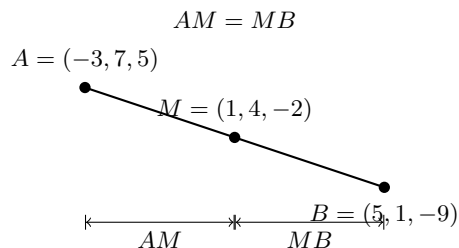
Key idea A vector separates two ideas: its components describe displacement, while its magnitude and unit vector describe size and direction. Always check signs and confirm that a unit vector has magnitude 1.

Question 6

Show that $M = (1, 4, -2)$ is the midpoint of the segment joining $A = (-3, 7, 5)$ and an unknown point B , and determine B .

Question 6 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 The midpoint equation is $M = (A + B)/2$, so $B = 2M - A$.

Step 2 Compute $2M = \langle 2, 8, -4 \rangle$ and subtract A .

Step 3 Thus $B = \boxed{(5, 1, -9)}$. Averaging A and B returns $(1, 4, -2)$, which verifies the claim.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

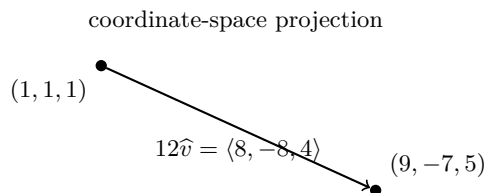
Key idea A vector separates two ideas: its components describe displacement, while its magnitude and unit vector describe size and direction. Always check signs and confirm that a unit vector has magnitude 1.

Question 7

A particle moves from $(1, 1, 1)$ in the direction $\langle 2, -2, 1 \rangle$. At what point has it traveled exactly 12 units?

Question 7 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 The direction magnitude is $\sqrt{4 + 4 + 1} = 3$, so its unit vector is $\frac{1}{3}\langle 2, -2, 1 \rangle$.

Step 2 A 12-unit displacement is $12\hat{v} = \langle 8, -8, 4 \rangle$.

Step 3 Adding the starting point gives $\boxed{(9, -7, 5)}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

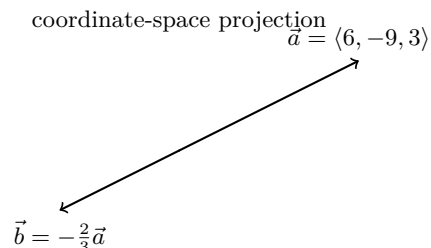
Key idea A vector separates two ideas: its components describe displacement, while its magnitude and unit vector describe size and direction. Always check signs and confirm that a unit vector has magnitude 1.

Question 8

Determine whether $\vec{a} = \langle 6, -9, 3 \rangle$ and $\vec{b} = \langle -4, 6, -2 \rangle$ represent the same direction, opposite directions, or neither. Explain.

Question 8 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 Compare corresponding nonzero components: $-4/6 = 6/(-9) = (-2)/3 = -2/3$.

Step 2 Thus $\vec{b} = (-2/3)\vec{a}$.

Step 3 Because the scalar is negative, the vectors are parallel but point in opposite directions.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

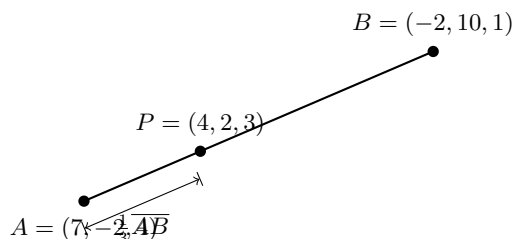
Key idea A vector separates two ideas: its components describe displacement, while its magnitude and unit vector describe size and direction. Always check signs and confirm that a unit vector has magnitude 1.

Question 9

Find the point one-third of the way from $A = (7, -2, 4)$ to $B = (-2, 10, 1)$.

Question 9 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 The displacement from A to B is $B - A = \langle -9, 12, -3 \rangle$.

Step 2 One-third of this displacement is $\langle -3, 4, -1 \rangle$.

Step 3 Starting at A gives $A + \frac{1}{3}(B - A) = \boxed{(4, 2, 3)}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

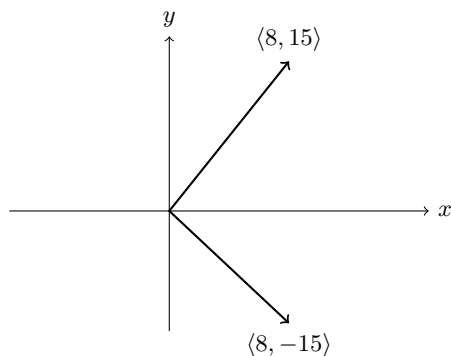
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Question 10

A velocity vector has horizontal component 8 and total speed 17. Find all possible vertical components and interpret the two answers.

Question 10 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 Let the vertical component be v . The speed condition is $\sqrt{8^2 + v^2} = 17$.

Step 2 Squaring yields $v^2 = 289 - 64 = 225$, so $v = \pm 15$.

Step 3 The velocities are $\boxed{\langle 8, 15 \rangle}$ and $\boxed{\langle 8, -15 \rangle}$, representing upward and downward motion with the same speed.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

Key idea A vector separates two ideas: its components describe displacement, while its magnitude and unit vector describe size and direction. Always check signs and confirm that a unit vector has magnitude 1.