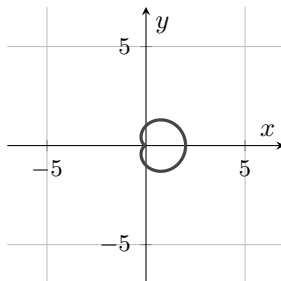
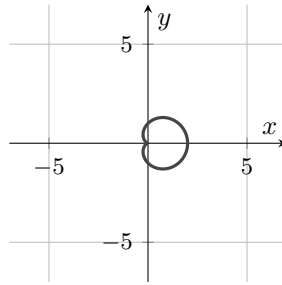


Question 1**Problem**

Find the length of the cardioid $r = 1 + \cos \theta$.



Question 1 – Solution**Solution**

1. Differentiate the polar radius to obtain $r' = dr/d\theta$ and choose an interval that traces the requested arc exactly once.
2. Use the polar arc-length formula

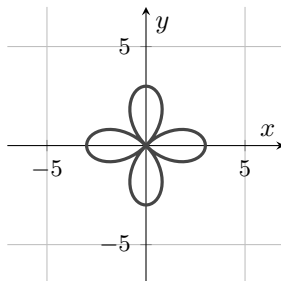
$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

Simplify the expression under the square root before evaluating or reporting the integral.

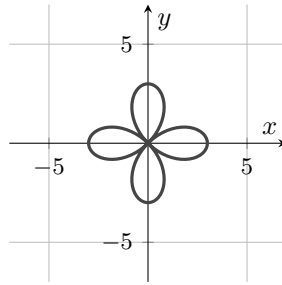
3. The speed simplifies to $2|\cos(\theta/2)|$.
4. Integrating from 0 to 2π gives $\boxed{8}$.

Question 2**Problem**

Find the length of one petal of $r = 3 \cos 2\theta$.



Question 2 – Solution

**Solution**

1. Differentiate the polar radius to obtain $r' = dr/d\theta$ and choose an interval that traces the requested arc exactly once.
2. Use the polar arc-length formula

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

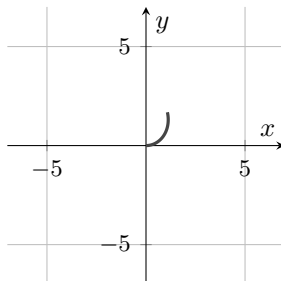
Simplify the expression under the square root before evaluating or reporting the integral.

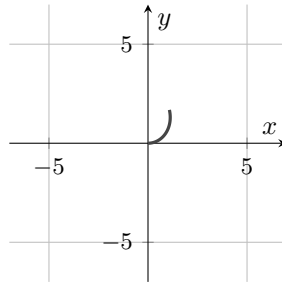
3. On $[-\pi/4, \pi/4]$, speed is $3\sqrt{\cos^2 2\theta + 4\sin^2 2\theta}$.

4. Thus $L = 3 \int_{-\pi/4}^{\pi/4} \sqrt{1 + 3\sin^2 2\theta} d\theta$; it is an elliptic integral.

Question 3**Problem**

Find the length of the spiral $r = 2\theta$, $0 \leq \theta \leq 1$.



Question 3 – Solution**Solution**

1. Differentiate the polar radius to obtain $r' = dr/d\theta$ and choose an interval that traces the requested arc exactly once.
2. Use the polar arc-length formula

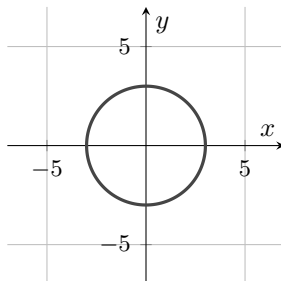
$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

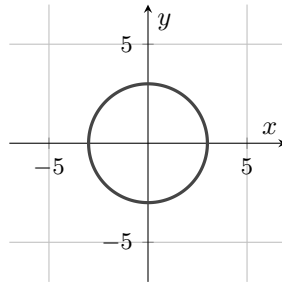
Simplify the expression under the square root before evaluating or reporting the integral.

3. $L = 2 \int_0^1 \sqrt{1 + \theta^2} d\theta = \boxed{\sqrt{2} + \operatorname{arsinh} 1}.$

Question 4**Problem**

Use the polar arc-length formula to recover the circumference of $r = R$.



Question 4 – Solution**Solution**

1. Differentiate the polar radius to obtain $r' = dr/d\theta$ and choose an interval that traces the requested arc exactly once.
2. Use the polar arc-length formula

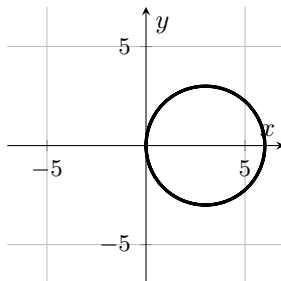
$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

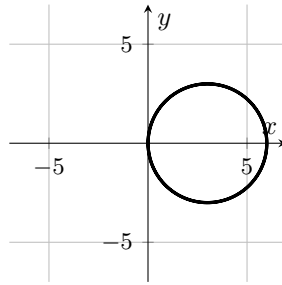
Simplify the expression under the square root before evaluating or reporting the integral.

3. Since $r' = 0$, speed is R .
4. Hence $L = \int_0^{2\pi} R d\theta = \boxed{2\pi R}$.

Question 5**Problem**

A circle of the same radius is $r = 2R \cos \theta$. Which interval traces it once, and what length results?



Question 5 – Solution**Solution**

1. Differentiate the polar radius to obtain $r' = dr/d\theta$ and choose an interval that traces the requested arc exactly once.
2. Use the polar arc-length formula

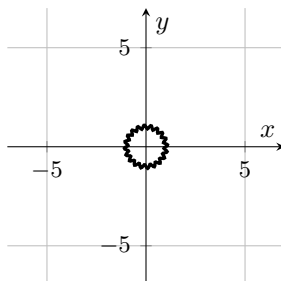
$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

Simplify the expression under the square root before evaluating or reporting the integral.

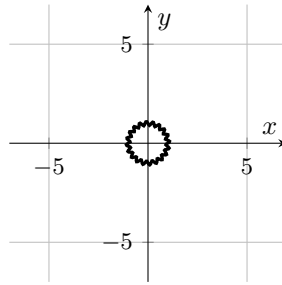
3. $-\pi/2 \leq \theta \leq \pi/2$ traces it once.
4. The speed simplifies to $2R$, so $L = 2R\pi = \boxed{2\pi R}$.

Question 6**Problem**

Without evaluating, compare the lengths of $r = 1$ and $r = 1 + 0.1 \sin(20\theta)$ over $0 \leq \theta \leq 2\pi$.



Question 6 – Solution

**Solution**

1. Differentiate the polar radius to obtain $r' = dr/d\theta$ and choose an interval that traces the requested arc exactly once.
2. Use the polar arc-length formula

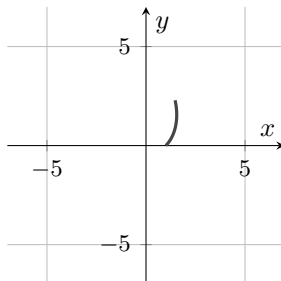
$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

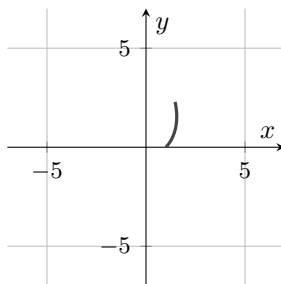
Simplify the expression under the square root before evaluating or reporting the integral.

3. Here $r > 0$ and $r' = 2 \cos 20\theta$. Thus $\sqrt{r^2 + (r')^2} \geq r$, with strict inequality except at finitely many angles. Consequently $L > \int_0^{2\pi} (1 + 0.1 \sin 20\theta) d\theta = 2\pi$, the unit circle's length.

Question 7**Problem**

Find the length of $r = e^\theta$, $0 \leq \theta \leq \ln 2$.



Question 7 – Solution**Solution**

1. Differentiate the polar radius to obtain $r' = dr/d\theta$ and choose an interval that traces the requested arc exactly once.
2. Use the polar arc-length formula

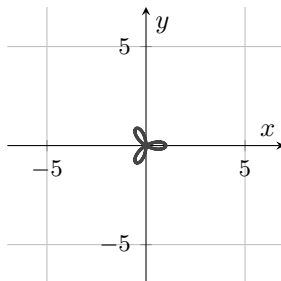
$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

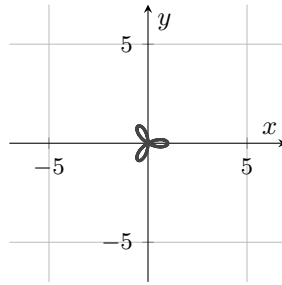
Simplify the expression under the square root before evaluating or reporting the integral.

3. Since $r' = r$, speed is $\sqrt{2}e^\theta$.
4. Thus $L = \sqrt{2}[e^\theta]_0^{\ln 2} = \boxed{\sqrt{2}}$.

Question 8**Problem**

Explain why integrating the rose $r = \cos 3\theta$ from 0 to 2π overcounts its geometric length.



Question 8 – Solution**Solution**

1. Differentiate the polar radius to obtain $r' = dr/d\theta$ and choose an interval that traces the requested arc exactly once.
2. Use the polar arc-length formula

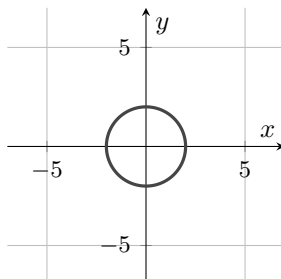
$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

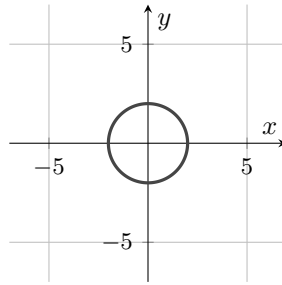
Simplify the expression under the square root before evaluating or reporting the integral.

3. For odd 3, the full curve is completed on $0 \leq \theta \leq \pi$; the next π retraces it because negative radius duplicates opposite points.
4. The 0 to 2π integral counts every petal twice.

Question 9**Problem**

A polar path has constant arc-length speed 5 and runs for an angular interval of width $2\pi/3$. What is its length?



Question 9 – Solution**Solution**

1. Differentiate the polar radius to obtain $r' = dr/d\theta$ and choose an interval that traces the requested arc exactly once.
2. Use the polar arc-length formula

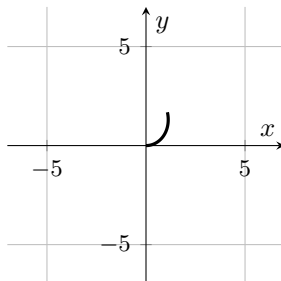
$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

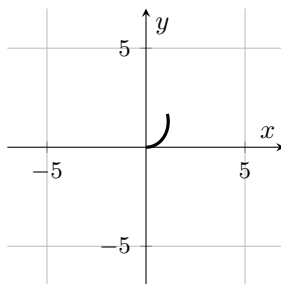
Simplify the expression under the square root before evaluating or reporting the integral.

3. Arc length is the integral of speed: $L = 5(2\pi/3) = \boxed{10\pi/3}$.

Question 10**Problem**

Find a so that $r = a\theta$, $0 \leq \theta \leq 1$, has length $\sqrt{2} + \operatorname{arsinh} 1$.



Question 10 – Solution**Solution**

1. Differentiate the polar radius to obtain $r' = dr/d\theta$ and choose an interval that traces the requested arc exactly once.
2. Use the polar arc-length formula

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

Simplify the expression under the square root before evaluating or reporting the integral.

3. Its length is $|a| \int_0^1 \sqrt{1 + \theta^2} d\theta = \frac{|a|}{2}(\sqrt{2} + \operatorname{arsinh} 1)$.
4. Thus $\boxed{a = \pm 2}$.