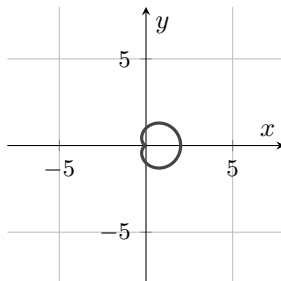
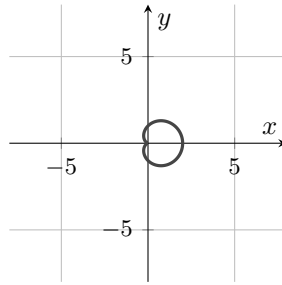


**Question 1****Problem**

Find the area enclosed by the cardioid  $r = 1 + \cos \theta$ .



**Question 1 – Solution****Solution**

1. Determine an interval of angles that sweeps out the requested region exactly once, using symmetry when it simplifies the work.
2. For a polar boundary  $r = f(\theta)$ , use

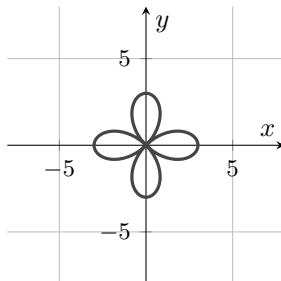
$$A = \frac{1}{2} \int_a^b r^2 d\theta.$$

For a region between two curves, subtract the inner squared radius from the outer squared radius before integrating.

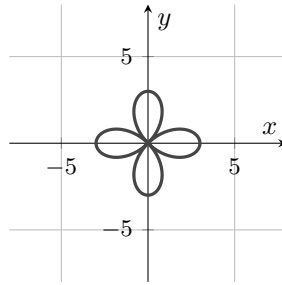
3.  $A = \frac{1}{2} \int_0^{2\pi} (1 + \cos \theta)^2 d\theta = \boxed{3\pi/2}.$

**Question 2****Problem**

Find the area of one petal of  $r = 3 \cos 2\theta$ .



## Question 2 – Solution

**Solution**

1. Determine an interval of angles that sweeps out the requested region exactly once, using symmetry when it simplifies the work.
2. For a polar boundary  $r = f(\theta)$ , use

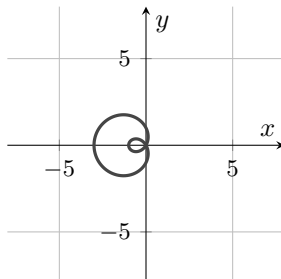
$$A = \frac{1}{2} \int_a^b r^2 d\theta.$$

For a region between two curves, subtract the inner squared radius from the outer squared radius before integrating.

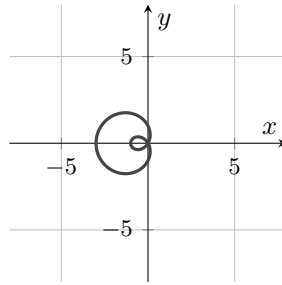
3. One petal is traced on  $[-\pi/4, \pi/4]$ .
4. Thus  $A = \frac{1}{2} \int_{-\pi/4}^{\pi/4} 9 \cos^2 2\theta d\theta = \boxed{9\pi/8}$ .

**Question 3****Problem**

Find the area of the inner loop of  $r = 1 - 2 \cos \theta$ .



## Question 3 – Solution

**Solution**

1. Determine an interval of angles that sweeps out the requested region exactly once, using symmetry when it simplifies the work.
2. For a polar boundary  $r = f(\theta)$ , use

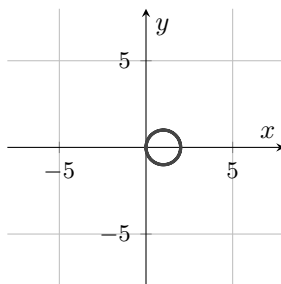
$$A = \frac{1}{2} \int_a^b r^2 d\theta.$$

For a region between two curves, subtract the inner squared radius from the outer squared radius before integrating.

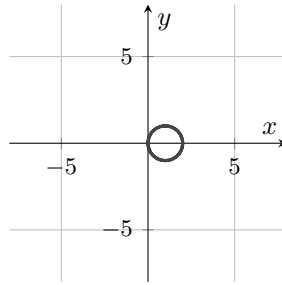
3. The loop is traced for  $-\pi/3 \leq \theta \leq \pi/3$ .
4. Hence  $A = \frac{1}{2} \int_{-\pi/3}^{\pi/3} (1 - 2 \cos \theta)^2 d\theta = \boxed{\pi - 3\sqrt{3}/2}$ .

**Question 4****Problem**

Find the common area of the circles  $r = 2 \cos \theta$  and  $r = 2 \sin \theta$ .



## Question 4 – Solution

**Solution**

1. Determine an interval of angles that sweeps out the requested region exactly once, using symmetry when it simplifies the work.
2. For a polar boundary  $r = f(\theta)$ , use

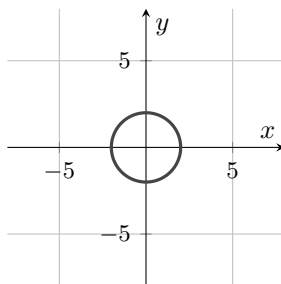
$$A = \frac{1}{2} \int_a^b r^2 d\theta.$$

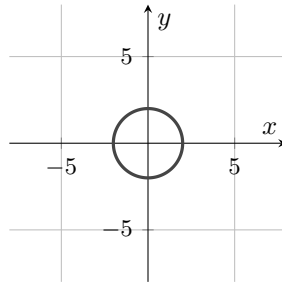
For a region between two curves, subtract the inner squared radius from the outer squared radius before integrating.

3. In the first quadrant the smaller radius switches at  $\pi/4$ .
4. Symmetry gives  $A = \int_0^{\pi/4} (2 \sin \theta)^2 d\theta = \boxed{\pi/2 - 1}$ .

**Question 5****Problem**

Find the area inside  $r = 2$  but outside  $r = 2 \cos \theta$ .



**Question 5 – Solution****Solution**

1. Determine an interval of angles that sweeps out the requested region exactly once, using symmetry when it simplifies the work.
2. For a polar boundary  $r = f(\theta)$ , use

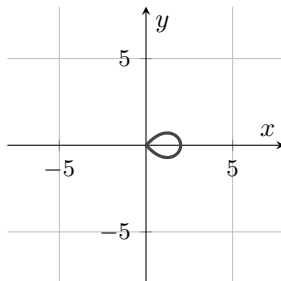
$$A = \frac{1}{2} \int_a^b r^2 d\theta.$$

For a region between two curves, subtract the inner squared radius from the outer squared radius before integrating.

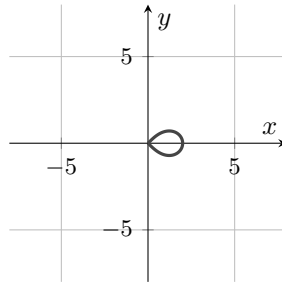
3. The circle  $r = 2 \cos \theta$  lies entirely inside  $r = 2$ .
4. Subtract areas:  $4\pi - \pi = \boxed{3\pi}$ .

**Question 6****Problem**

The curve  $r^2 = 4 \cos 2\theta$  has two loops. Find its total area.



## Question 6 – Solution

**Solution**

1. Determine an interval of angles that sweeps out the requested region exactly once, using symmetry when it simplifies the work.
2. For a polar boundary  $r = f(\theta)$ , use

$$A = \frac{1}{2} \int_a^b r^2 d\theta.$$

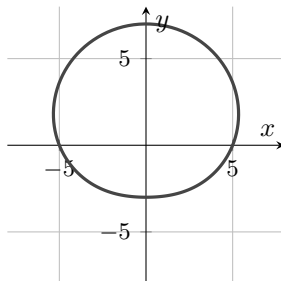
For a region between two curves, subtract the inner squared radius from the outer squared radius before integrating.

3. One loop uses  $-\pi/4 \leq \theta \leq \pi/4$ :  $\text{area} = \frac{1}{2} \int 4 \cos 2\theta d\theta = 2$ .
4. Doubling gives 4.

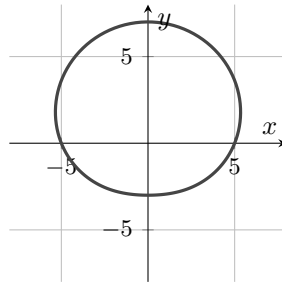
## Question 7

## Problem

A radar beam reaches distance  $r = 5 + 2 \sin \theta$ . Find the area swept while  $0 \leq \theta \leq \pi$ .



## Question 7 – Solution



## Solution

1. Determine an interval of angles that sweeps out the requested region exactly once, using symmetry when it simplifies the work.
2. For a polar boundary  $r = f(\theta)$ , use

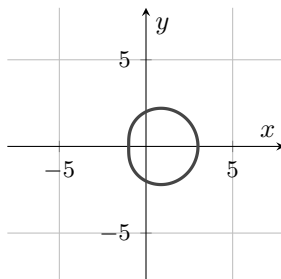
$$A = \frac{1}{2} \int_a^b r^2 d\theta.$$

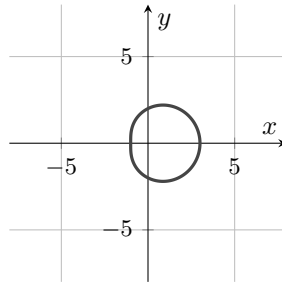
For a region between two curves, subtract the inner squared radius from the outer squared radius before integrating.

3.  $A = \frac{1}{2} \int_0^\pi (5 + 2 \sin \theta)^2 d\theta = \boxed{27\pi/2 + 20}.$

**Question 8****Problem**

Two polar curves satisfy  $0 \leq r_1(\theta) \leq r_2(\theta)$ . Explain why the area between them involves squares, not merely  $r_2 - r_1$ .



**Question 8 – Solution****Solution**

1. Determine an interval of angles that sweeps out the requested region exactly once, using symmetry when it simplifies the work.
2. For a polar boundary  $r = f(\theta)$ , use

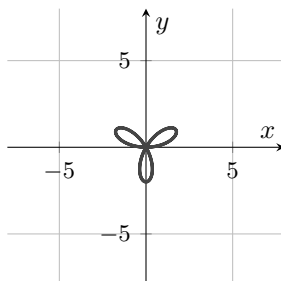
$$A = \frac{1}{2} \int_a^b r^2 d\theta.$$

For a region between two curves, subtract the inner squared radius from the outer squared radius before integrating.

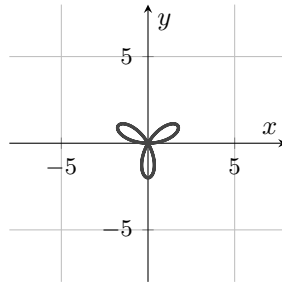
3. A thin polar sector has area  $\frac{1}{2}r^2 d\theta$ .
4. Subtracting sector areas yields  $\frac{1}{2}(r_2^2 - r_1^2)d\theta$ , reflecting increasing width with radius.

**Question 9****Problem**

Find the area enclosed by one loop of  $r = 2 \sin 3\theta$  and explain why multiplying by six is wrong for total area.



## Question 9 – Solution

**Solution**

1. Determine an interval of angles that sweeps out the requested region exactly once, using symmetry when it simplifies the work.
2. For a polar boundary  $r = f(\theta)$ , use

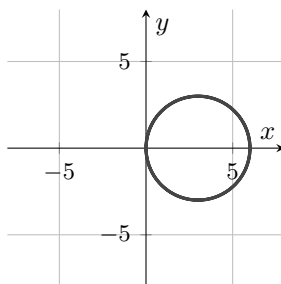
$$A = \frac{1}{2} \int_a^b r^2 d\theta.$$

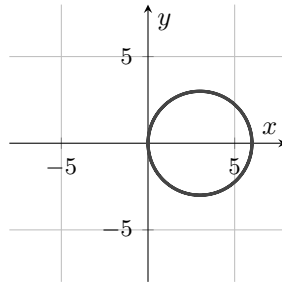
For a region between two curves, subtract the inner squared radius from the outer squared radius before integrating.

3. One loop area is  $\frac{1}{2} \int_0^{\pi/3} 4 \sin^2 3\theta d\theta = \pi/3$ .
4. The rose has three petals, not six, because negative radii retrace opposite petals; total area is  $\boxed{\pi}$ .

**Question 10****Problem**

Choose  $a > 0$  so that the circle  $r = a \cos \theta$  has area  $9\pi$ .



**Question 10 – Solution****Solution**

1. Determine an interval of angles that sweeps out the requested region exactly once, using symmetry when it simplifies the work.
2. For a polar boundary  $r = f(\theta)$ , use

$$A = \frac{1}{2} \int_a^b r^2 d\theta.$$

For a region between two curves, subtract the inner squared radius from the outer squared radius before integrating.

3. It is a circle of radius  $a/2$ , so area is  $\pi a^2/4$ .
4. Setting this to  $9\pi$  gives  $\boxed{a = 6}$ .