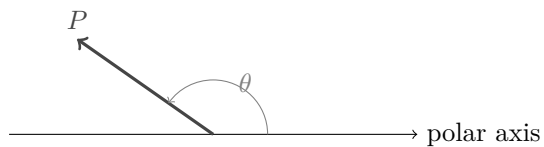
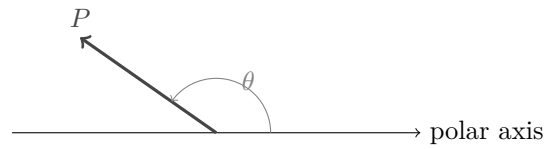


Question 1**Problem**

The point has polar coordinates $(-4, \pi/3)$. Give two equivalent representations with positive radius and locate its quadrant.



Question 1 – Solution**Solution**

1. Use the polar–Cartesian relationships

$$x = r \cos \theta, \quad y = r \sin \theta, \quad r^2 = x^2 + y^2.$$

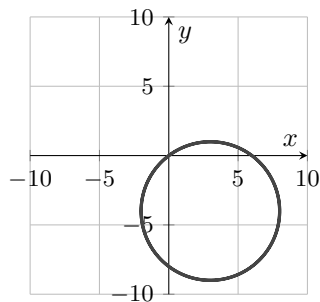
Equivalent polar coordinates satisfy

$$(r, \theta) = (r, \theta + 2k\pi) = (-r, \theta + (2k + 1)\pi).$$

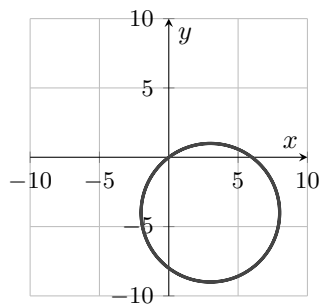
2. Apply the identity that matches the requested conversion, symmetry test, or intersection, and then check the resulting point or curve in the original polar equation.
3. Adding π while changing the radius sign gives $(4, 4\pi/3)$; also $(4, -2\pi/3)$.
4. It lies in Quadrant III.

Question 2**Problem**

Convert $r = 6 \cos \theta - 8 \sin \theta$ to Cartesian form and identify the curve.



Question 2 – Solution

**Solution**

1. Use the polar–Cartesian relationships

$$x = r \cos \theta, \quad y = r \sin \theta, \quad r^2 = x^2 + y^2.$$

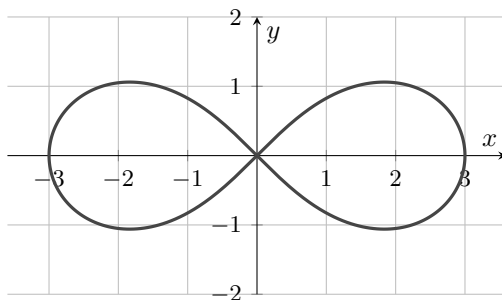
Equivalent polar coordinates satisfy

$$(r, \theta) = (r, \theta + 2k\pi) = (-r, \theta + (2k + 1)\pi).$$

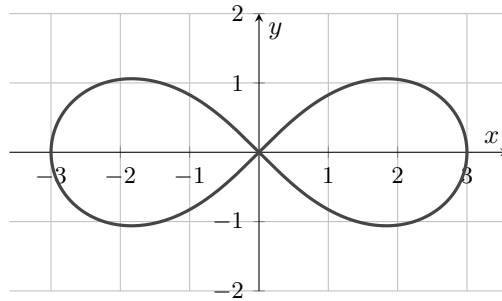
2. Apply the identity that matches the requested conversion, symmetry test, or intersection, and then check the resulting point or curve in the original polar equation.
3. $r^2 = 6x - 8y$, so $(x - 3)^2 + (y + 4)^2 = 25$: a circle centered at $(3, -4)$ with radius 5.

Question 3**Problem**

Convert $r^2 = 9 \cos 2\theta$ to Cartesian form without using inverse trigonometric functions.



Question 3 – Solution

**Solution**

1. Use the polar–Cartesian relationships

$$x = r \cos \theta, \quad y = r \sin \theta, \quad r^2 = x^2 + y^2.$$

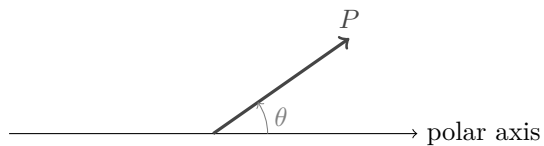
Equivalent polar coordinates satisfy

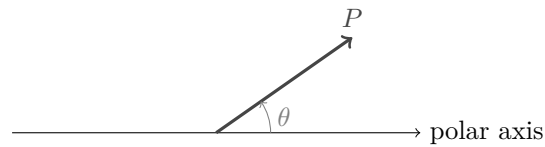
$$(r, \theta) = (r, \theta + 2k\pi) = (-r, \theta + (2k + 1)\pi).$$

2. Apply the identity that matches the requested conversion, symmetry test, or intersection, and then check the resulting point or curve in the original polar equation.
3. Since $\cos 2\theta = (x^2 - y^2)/r^2$, $r^4 = 9(x^2 - y^2)$.
4. Thus $\boxed{(x^2 + y^2)^2 = 9(x^2 - y^2)}$.

Question 4**Problem**

Determine all intersections of $r = 2$ and $r = 4 \cos \theta$.



Question 4 – Solution**Solution**

1. Use the polar–Cartesian relationships

$$x = r \cos \theta, \quad y = r \sin \theta, \quad r^2 = x^2 + y^2.$$

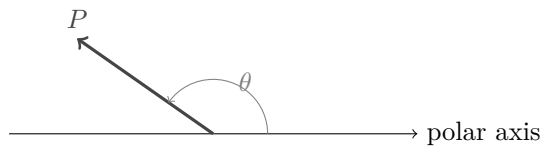
Equivalent polar coordinates satisfy

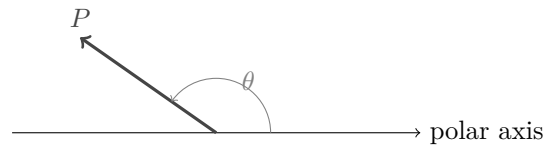
$$(r, \theta) = (r, \theta + 2k\pi) = (-r, \theta + (2k + 1)\pi).$$

2. Apply the identity that matches the requested conversion, symmetry test, or intersection, and then check the resulting point or curve in the original polar equation.
3. $2 = 4 \cos \theta$ gives $\theta = \pm\pi/3$.
4. The points are $(1, \pm\sqrt{3})$.

Question 5**Problem**

A polar curve is unchanged when θ is replaced by $-\theta$. What symmetry does this reveal, and why?



Question 5 – Solution**Solution**

1. Use the polar–Cartesian relationships

$$x = r \cos \theta, \quad y = r \sin \theta, \quad r^2 = x^2 + y^2.$$

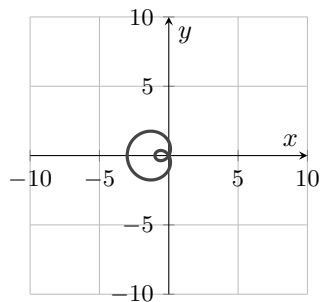
Equivalent polar coordinates satisfy

$$(r, \theta) = (r, \theta + 2k\pi) = (-r, \theta + (2k + 1)\pi).$$

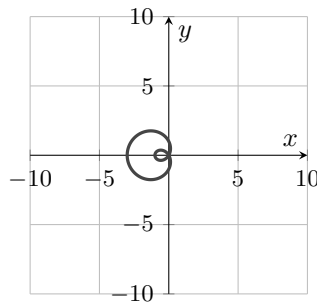
2. Apply the identity that matches the requested conversion, symmetry test, or intersection, and then check the resulting point or curve in the original polar equation.
3. The points at angles θ and $-\theta$ reflect across the polar axis, so the curve is symmetric about the x -axis.

Question 6**Problem**

Sketch mentally $r = 1 - 2 \cos \theta$. Why does it have an inner loop?



Question 6 – Solution

**Solution**

1. Use the polar–Cartesian relationships

$$x = r \cos \theta, \quad y = r \sin \theta, \quad r^2 = x^2 + y^2.$$

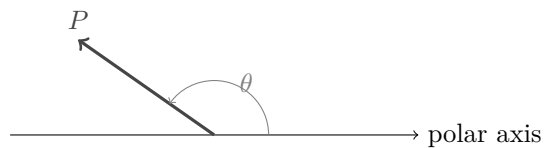
Equivalent polar coordinates satisfy

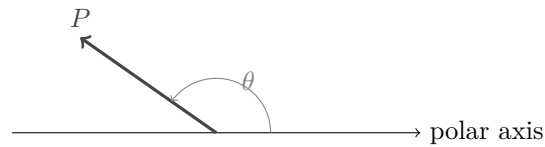
$$(r, \theta) = (r, \theta + 2k\pi) = (-r, \theta + (2k + 1)\pi).$$

2. Apply the identity that matches the requested conversion, symmetry test, or intersection, and then check the resulting point or curve in the original polar equation.
3. The radius becomes negative when $\cos \theta > 1/2$, i.e. near $\theta = 0$.
4. Negative radii plot in the opposite direction, producing the inner loop.

Question 7**Problem**

Find the Cartesian equation corresponding to $\theta = 2\pi/3$ and explain why r must be unrestricted.



Question 7 – Solution**Solution**

1. Use the polar–Cartesian relationships

$$x = r \cos \theta, \quad y = r \sin \theta, \quad r^2 = x^2 + y^2.$$

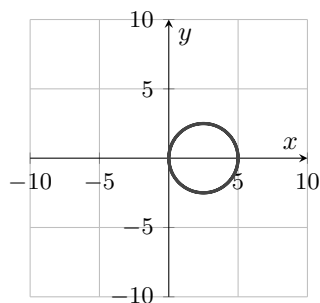
Equivalent polar coordinates satisfy

$$(r, \theta) = (r, \theta + 2k\pi) = (-r, \theta + (2k + 1)\pi).$$

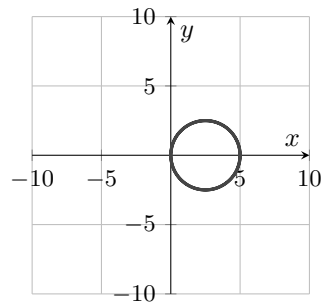
2. Apply the identity that matches the requested conversion, symmetry test, or intersection, and then check the resulting point or curve in the original polar equation.
3. The line has slope $\tan(2\pi/3) = -\sqrt{3}$, so $y = -\sqrt{3}x$.
4. Allowing negative r supplies the opposite ray and therefore the entire line.

Question 8**Problem**

Which description better reveals the geometry of $x^2 + y^2 = 5x$: Cartesian or polar? Convert it and explain.



Question 8 – Solution

**Solution**

1. Use the polar–Cartesian relationships

$$x = r \cos \theta, \quad y = r \sin \theta, \quad r^2 = x^2 + y^2.$$

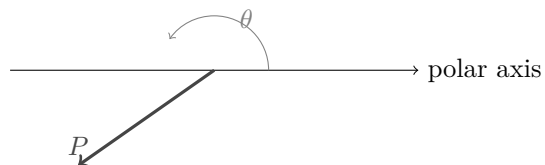
Equivalent polar coordinates satisfy

$$(r, \theta) = (r, \theta + 2k\pi) = (-r, \theta + (2k + 1)\pi).$$

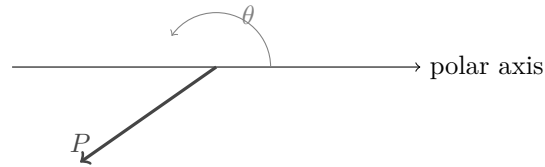
2. Apply the identity that matches the requested conversion, symmetry test, or intersection, and then check the resulting point or curve in the original polar equation.
3. Polar form is $r = 5 \cos \theta$, immediately revealing a circle through the pole with diameter 5 along the polar axis.

Question 9**Problem**

Find the smallest positive angle between the rays representing $(3, 17\pi/12)$ and $(2, -\pi/4)$.



Question 9 – Solution



Solution

1. Use the polar–Cartesian relationships

$$x = r \cos \theta, \quad y = r \sin \theta, \quad r^2 = x^2 + y^2.$$

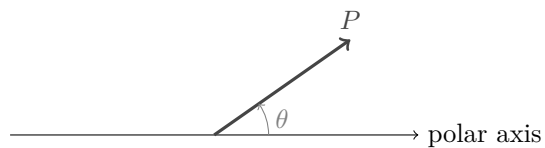
Equivalent polar coordinates satisfy

$$(r, \theta) = (r, \theta + 2k\pi) = (-r, \theta + (2k + 1)\pi).$$

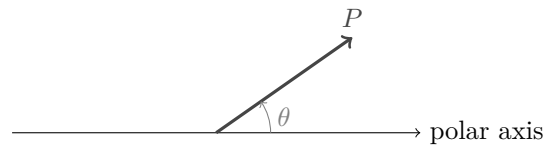
2. Apply the identity that matches the requested conversion, symmetry test, or intersection, and then check the resulting point or curve in the original polar equation.
3. Reduce angles to $17\pi/12$ and $21\pi/12$.
4. Their smaller difference is $\boxed{\pi/3}$.

Question 10**Problem**

A navigation system reports $(r, \theta) = (5, \text{atan2}(4, -3))$. Convert to Cartesian coordinates.



Question 10 – Solution

**Solution**

1. Use the polar–Cartesian relationships

$$x = r \cos \theta, \quad y = r \sin \theta, \quad r^2 = x^2 + y^2.$$

Equivalent polar coordinates satisfy

$$(r, \theta) = (r, \theta + 2k\pi) = (-r, \theta + (2k + 1)\pi).$$

2. Apply the identity that matches the requested conversion, symmetry test, or intersection, and then check the resulting point or curve in the original polar equation.
3. The angle is in Quadrant II with cosine $-3/5$ and sine $4/5$.
4. Hence $(x, y) = \boxed{(-3, 4)}$.