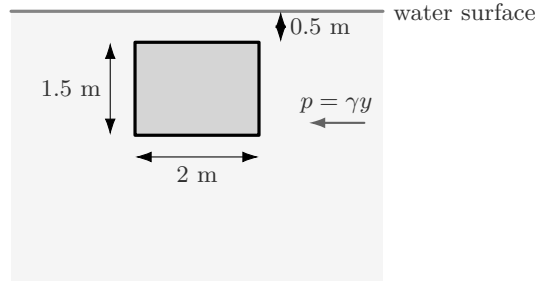
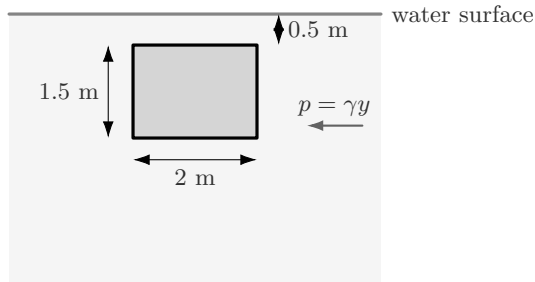


Question 1

A vertical aquarium window is (2) m wide and (1.5) m high; its top is (0.5) m below freshwater. Find the force ($\gamma = 9800 \text{ N/m}^3$).



Question 1 – Solution



Step 1: Choose the depth variable. Let y be the depth below the water surface, measured in meters. The window extends from

$$y = 0.5 \quad \text{to} \quad y = 0.5 + 1.5 = 2.$$

Step 2: Write the pressure at depth y . Freshwater has weight density $\gamma = 9800 \text{ N/m}^3$, so

$$p(y) = \gamma y = 9800y \quad \text{N/m}^2.$$

Step 3: Find the area of a horizontal strip. The window is 2 m wide. A strip of thickness dy has area

$$dA = 2 \, dy.$$

Therefore,

$$dF = p(y) \, dA = (9800y)(2 \, dy).$$

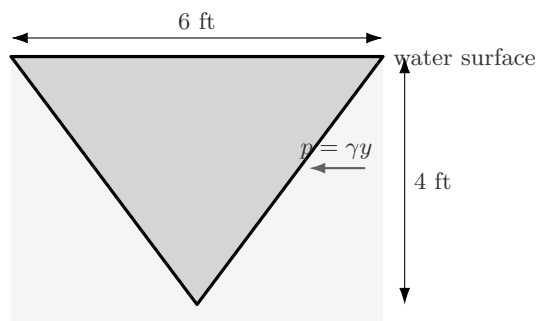
Step 4: Integrate over the window.

$$\begin{aligned} F &= 9800 \int_{0.5}^2 2y \, dy = 9800 [y^2]_{0.5}^2 \\ &= 9800(4 - 0.25) = 9800(3.75) = 36,750 \text{ N}. \end{aligned}$$

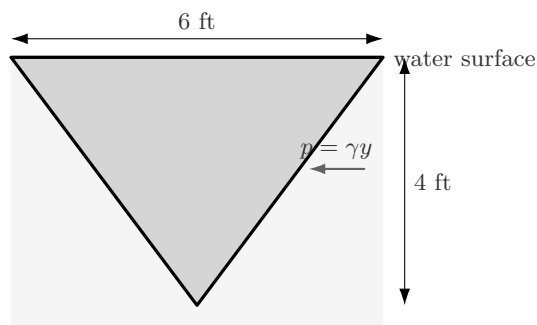
$$\boxed{F = 36,750 \text{ N}}$$

Question 2

An inverted triangular gate has top width (6) ft at the surface and vertex (4) ft deep. Find the force ($\gamma = 62.4 \text{ lb/ft}^3$).



Question 2 – Solution



Step 1: Choose the depth variable. Let y be depth below the water surface. The gate extends from $y = 0$ to $y = 4$ ft.

Step 2: Find the width of the gate at depth y . The width decreases linearly from 6 ft at $y = 0$ to 0 ft at $y = 4$. Its slope is

$$\frac{0 - 6}{4 - 0} = -\frac{3}{2},$$

so

$$w(y) = 6 - \frac{3}{2}y.$$

Step 3: Write the force on a horizontal strip. At depth y ,

$$p(y) = 62.4y \text{ lb/ft}^2, \quad dA = w(y) dy.$$

Thus

$$dF = 62.4y \left(6 - \frac{3}{2}y \right) dy.$$

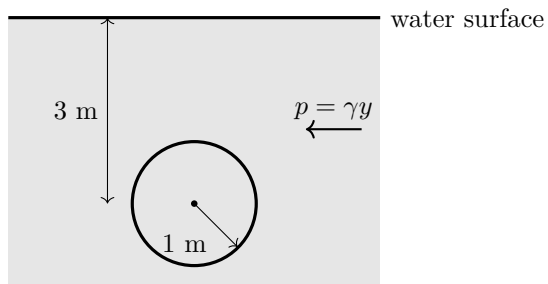
Step 4: Integrate and evaluate.

$$\begin{aligned} F &= 62.4 \int_0^4 y \left(6 - \frac{3}{2}y \right) dy \\ &= 62.4 \int_0^4 \left(6y - \frac{3}{2}y^2 \right) dy \\ &= 62.4 \left[3y^2 - \frac{1}{2}y^3 \right]_0^4 \\ &= 62.4(48 - 32) = 62.4(16) = 998.4 \text{ lb.} \end{aligned}$$

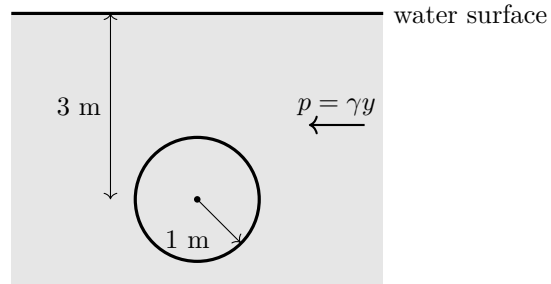
$$\boxed{F = 998.4 \text{ lb}}$$

Question 3

A vertical circular porthole of radius 1 m has center 3 m below the water surface. Use water weight density 9800 N/m^3 . Find the force without integrating strips.



Question 3 – Solution



Step 1: Use the centroid form of the hydrostatic-force formula. For a submerged vertical plane region,

$$F = \gamma A \bar{y},$$

where A is its area and $\bar{y}$ is the depth of its centroid.

Step 2: Find the area of the porthole. The radius is 1 m, so

$$A = \pi r^2 = \pi(1)^2 = \pi \text{ m}^2.$$

Step 3: Identify the centroid depth. The centroid of a circle is its center, which is 3 m below the surface. Therefore,

$$\bar{y} = 3 \text{ m}.$$

Step 4: Substitute.

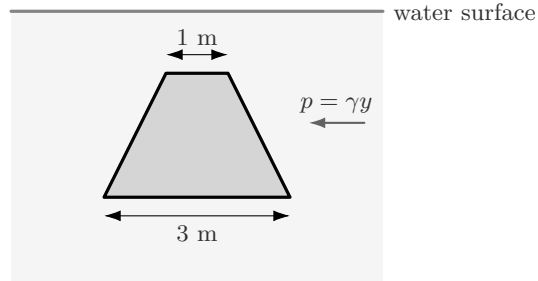
$$\begin{aligned} F &= (9800)(\pi)(3) \\ &= 29,400\pi \text{ N}. \end{aligned}$$

$$\boxed{F = 29,400\pi \text{ N}}$$

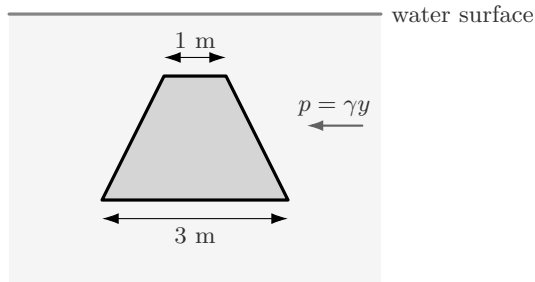
No strip integration is needed because the area and centroid depth are known.

Question 4

Oil has weight density 8000 N/m^3 . A trapezoidal plate is (1) m wide at its top, (3) m wide at its bottom, (2) m high, with top depth (1) m. Find the force.



Question 4 – Solution



Step 1: Choose the depth variable. Let y be depth below the oil surface. The plate begins at $y = 1$ and ends at $y = 3$ m.

Step 2: Find the width at depth y . The width grows linearly from 1 m at $y = 1$ to 3 m at $y = 3$. The slope is

$$\frac{3 - 1}{3 - 1} = 1,$$

so the linear width function is

$$w(y) = y, \quad 1 \leq y \leq 3.$$

Step 3: Write the force on a horizontal strip.

$$p(y) = 8000y, \quad dA = w(y) dy = y dy.$$

Hence

$$dF = p(y) dA = 8000y^2 dy.$$

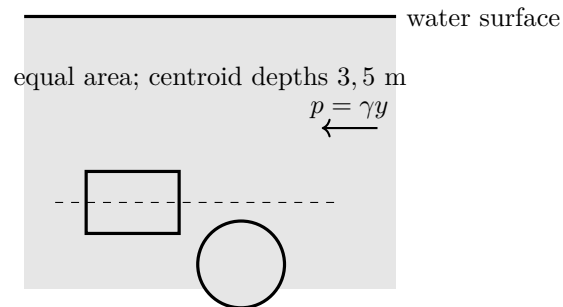
Step 4: Integrate and evaluate.

$$\begin{aligned} F &= 8000 \int_1^3 y^2 dy = 8000 \left[\frac{y^3}{3} \right]_1^3 \\ &= \frac{8000}{3} (27 - 1) = \frac{208000}{3} \text{ N}. \end{aligned}$$

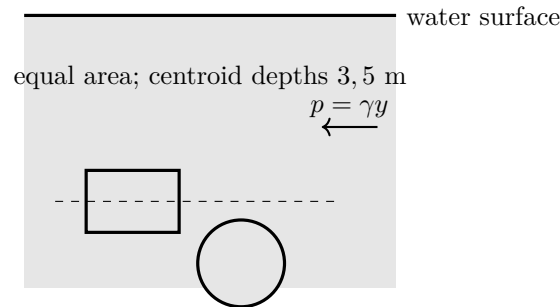
$$F = \frac{208000}{3} \text{ N} \approx 69,333.3 \text{ N}$$

Question 5

Two equal-area vertical plates are fully submerged in the same liquid of constant weight density, with centroid depths 3 m and 5 m. Compare their hydrostatic forces without knowing their shapes.



Question 5 – Solution



Step 1: Use the centroid formula. For any vertical plane region submerged in a liquid of constant weight density,

$$F = \gamma A \bar{y}.$$

Step 2: Write the force on each plate. Let the common area be A . If Plate 1 has centroid depth 3 m and Plate 2 has centroid depth 5 m, then

$$F_1 = \gamma A(3), \quad F_2 = \gamma A(5).$$

Step 3: Compare the forces.

$$\frac{F_2}{F_1} = \frac{\gamma A(5)}{\gamma A(3)} = \frac{5}{3}.$$

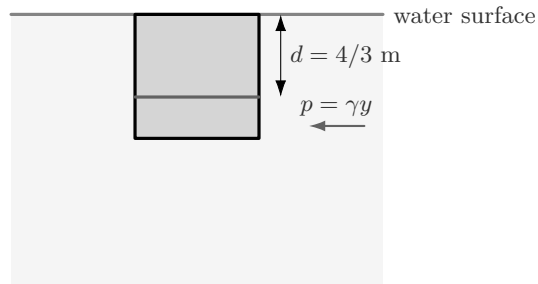
The shape does not affect this comparison because both plates have the same area; only the centroid depth differs.

$$F_2 = \frac{5}{3}F_1$$

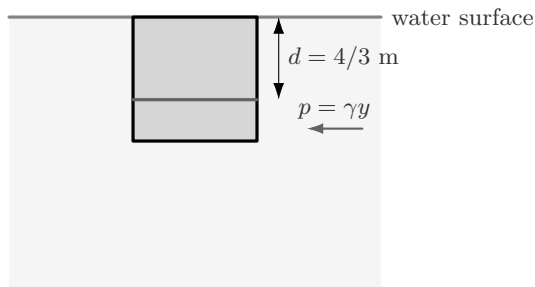
Thus the deeper plate experiences $66\frac{2}{3}\%$ more force.

Question 6

A 2×2 m square gate has its top at the surface. Find the depth d of the center of pressure, where $d = M/F$.



Question 6 – Solution



Step 1: Set up the hydrostatic force. Let y be depth below the surface. The gate has constant width 2 m and extends from $y = 0$ to $y = 2$. Thus

$$dF = (9800y)(2 dy).$$

Therefore,

$$\begin{aligned} F &= 9800 \int_0^2 2y \, dy = 19600 \left[\frac{y^2}{2} \right]_0^2 \\ &= 39,200 \text{ N.} \end{aligned}$$

Step 2: Find the moment of the pressure force about the surface. The moment arm of the strip force is y , so

$$\begin{aligned} M &= \int_0^2 y \, dF = 9800 \int_0^2 2y^2 \, dy \\ &= 19600 \left[\frac{y^3}{3} \right]_0^2 = \frac{156,800}{3} \text{ N m.} \end{aligned}$$

Step 3: Divide moment by total force.

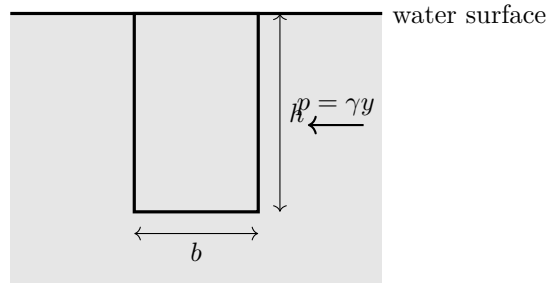
$$d = \frac{M}{F} = \frac{\frac{156800}{3}}{39200} = \frac{4}{3} \text{ m.}$$

$$d = \frac{4}{3} \text{ m below the surface}$$

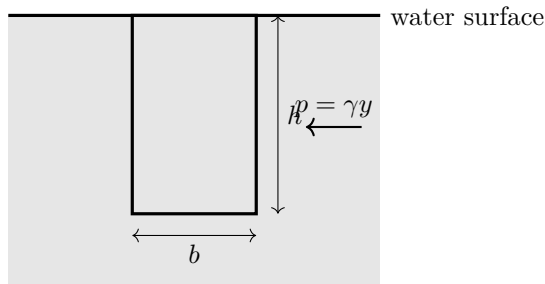
The center of pressure lies below the geometric center at depth 1 m because pressure increases with depth.

Question 7

A rectangular dam face is b meters wide and extends from the surface to depth h . Derive $F(h)$. If h doubles, by what factor does F change?



Question 7 – Solution



Step 1: Choose the depth variable. Let y measure depth below the surface. The dam face extends from $y = 0$ to $y = h$.

Step 2: Write the force on a horizontal strip. The pressure is $p(y) = \gamma y$, and a strip of width b and thickness dy has area $dA = b dy$. Therefore,

$$dF = p(y) dA = \gamma b y dy.$$

Step 3: Integrate from the surface to depth h .

$$\begin{aligned} F(h) &= \gamma b \int_0^h y dy \\ &= \gamma b \left[\frac{y^2}{2} \right]_0^h = \frac{1}{2} \gamma b h^2. \end{aligned}$$

$$F(h) = \frac{1}{2} \gamma b h^2$$

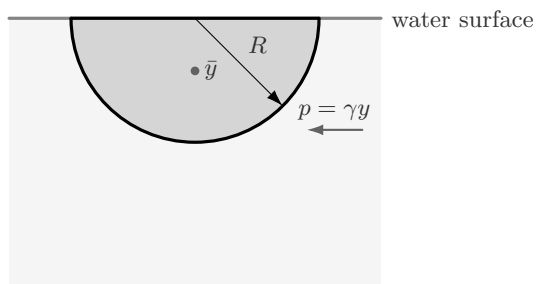
Step 4: Determine the effect of doubling the depth.

$$F(2h) = \frac{1}{2} \gamma b (2h)^2 = 4 \left(\frac{1}{2} \gamma b h^2 \right) = 4F(h).$$

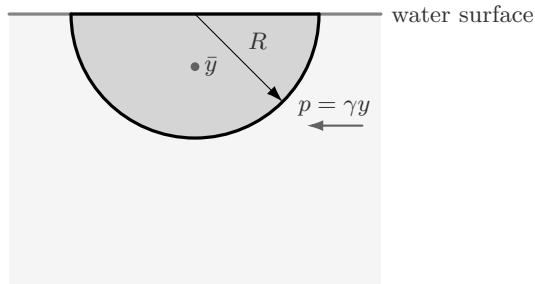
Doubling h multiplies the force by 4.

Question 8

A semicircular gate of radius R has its diameter along the water surface and lies below it. Find the force using centroid depth $4R/(3\pi)$.



Question 8 – Solution



Step 1: Use the centroid form of hydrostatic force. For a submerged vertical plane region,

$$F = \gamma A \bar{y}.$$

Step 2: Find the area of the semicircle.

$$A = \frac{1}{2}\pi R^2.$$

Step 3: Use the given centroid depth. Because the diameter lies on the water surface and the semicircle lies below it,

$$\bar{y} = \frac{4R}{3\pi}.$$

Step 4: Substitute and simplify.

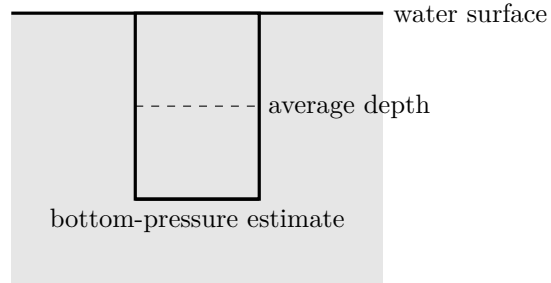
$$\begin{aligned} F &= \gamma \left(\frac{1}{2}\pi R^2 \right) \left(\frac{4R}{3\pi} \right) \\ &= \gamma \left(\frac{4R^3}{6} \right) = \frac{2}{3}\gamma R^3. \end{aligned}$$

$$\boxed{F = \frac{2}{3}\gamma R^3}$$

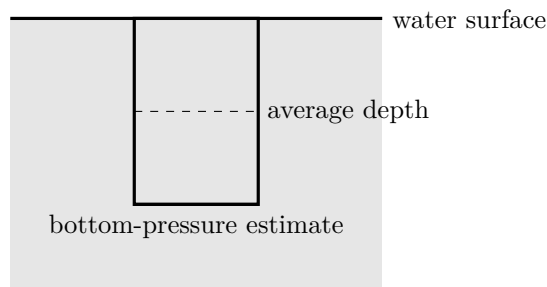
The factors of π cancel because the semicircle's area contains π while its centroid depth contains $1/\pi$.

Question 9

A student uses constant pressure at the bottom of a 3-m-tall vertical rectangular gate. Explain the resulting bias and find the ratio of this estimate to the true force when the top is at the surface.



Question 9 – Solution



Step 1: Write the student's estimate. The student applies the bottom pressure $p = \gamma(3)$ uniformly over the entire gate of area A . Thus

$$F_{\text{est}} = 3\gamma A.$$

Step 2: Find the true force. For a rectangular gate extending from the surface to depth 3 m, the centroid is at the average depth

$$\bar{y} = \frac{0 + 3}{2} = \frac{3}{2} \text{ m.}$$

Therefore,

$$F_{\text{true}} = \gamma A \bar{y} = \frac{3}{2} \gamma A.$$

Step 3: Compare the estimate with the true force.

$$\frac{F_{\text{est}}}{F_{\text{true}}} = \frac{3\gamma A}{\frac{3}{2}\gamma A} = 2.$$

Thus the estimate is twice the true value:

$$F_{\text{est}} = 2F_{\text{true}}.$$

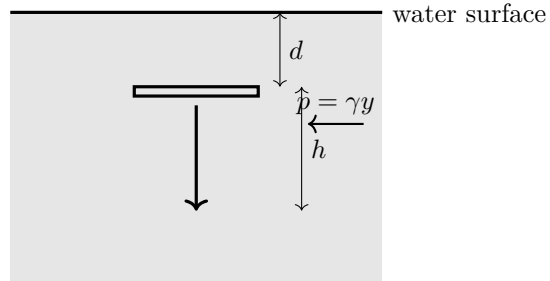
Step 4: Express the bias as a percent error.

$$\frac{F_{\text{est}} - F_{\text{true}}}{F_{\text{true}}} \times 100\% = (2 - 1)100\% = 100\%.$$

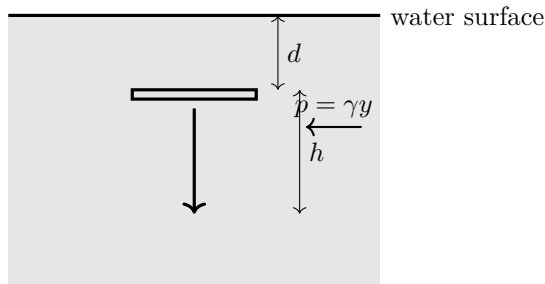
The bottom-pressure estimate is 100% too high.

Question 10

A horizontal hatch of constant area $A > 0$ is slowly lowered from depth d to $d + h$, where $d, h > 0$. Assume an upward resisting hydrostatic force $F(y) = \gamma Ay$ at depth y , with constant weight density $\gamma > 0$. Find the work done against this force, ignoring other forces.



Question 10 – Solution



Step 1: State the force model. Assume the hydrostatic force resisting the downward motion has magnitude

$$F(y) = p(y)A = \gamma yA,$$

where y is the hatch depth. The area A is constant.

Step 2: Write a differential amount of work. Moving the hatch through a small downward distance dy requires

$$dW = F(y) dy = \gamma Ay dy.$$

Step 3: Integrate over the change in depth. The hatch moves from $y = d$ to $y = d + h$, so

$$\begin{aligned} W &= \int_d^{d+h} \gamma Ay dy \\ &= \gamma A \left[\frac{y^2}{2} \right]_d^{d+h} \\ &= \frac{\gamma A}{2} ((d+h)^2 - d^2). \end{aligned}$$

Step 4: Simplify.

$$\begin{aligned} W &= \frac{\gamma A}{2} (d^2 + 2dh + h^2 - d^2) \\ &= \gamma A \left(dh + \frac{h^2}{2} \right). \end{aligned}$$

$$\boxed{W = \gamma A \left(dh + \frac{h^2}{2} \right)}$$

This is the external work under the stated resisting-force assumption.