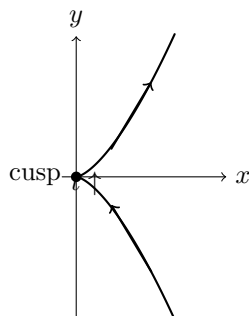


Question 10

Consider $\vec{r}(t) = \langle t^2, t^3 \rangle$ on $-1 \leq t \leq 1$. The parameterization is not regular at $t = 0$. Determine the total arc length exactly and explain why one must split the integral or preserve an absolute value.

Question 10 – Solution

Strategy The speed contains $\sqrt{t^2} = |t|$; use symmetry across the cusp.



Speed

$$\vec{r}'(t) = \langle 2t, 3t^2 \rangle, \quad \|\vec{r}'(t)\| = |t|\sqrt{4 + 9t^2}.$$

Length

$$L = 2 \int_0^1 t\sqrt{4 + 9t^2} dt = \frac{2}{27} \left[(4 + 9t^2)^{3/2} \right]_0^1 = \boxed{\frac{2}{27}(13\sqrt{13} - 8)}.$$

Why care about $|t|$? Replacing $|t|$ by t on the full symmetric interval would make the integrand odd and falsely give zero. Although velocity vanishes at the cusp, each half is smooth and their nonnegative lengths add.