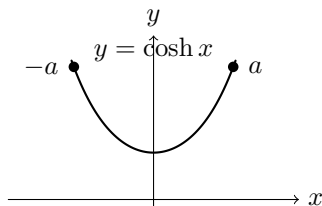


Question 9

The catenary $\vec{r}(t) = \langle t, \cosh t \rangle$ is restricted to $-a \leq t \leq a$, where $a > 0$. Find its exact length in terms of a . Then determine the value of a that makes the length equal to 6.

Question 9 – Solution

Strategy Use the identity $1 + \sinh^2 t = \cosh^2 t$ and the positivity of $\cosh t$.



Length

$$\|\vec{r}'(t)\| = \sqrt{1 + \sinh^2 t} = \cosh t, \quad L = \int_{-a}^a \cosh t \, dt = 2 \sinh a.$$

Therefore $\boxed{L = 2 \sinh a}$.

Prescribed length Setting $2 \sinh a = 6$ gives $\sinh a = 3$, hence

$$\boxed{a = \operatorname{arsinh}(3) = \ln(3 + \sqrt{10})}.$$

Symmetry The even speed makes the two halves have equal length; equivalently, $L = 2 \int_0^a \cosh t \, dt$.