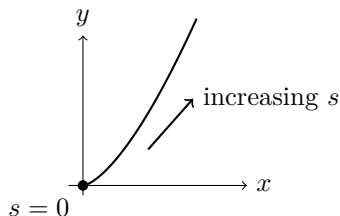


Question 7

For $t \geq 0$, let $\vec{r}(t) = \langle \frac{1}{2}t^2, \frac{1}{3}t^3 \rangle$. Construct the arc-length coordinate measured from $t = 0$, invert it explicitly, and give a unit-speed parameterization $\vec{R}(s)$.

Question 7 – Solution

Strategy The factor $t \geq 0$ makes the speed easy to integrate; then solve algebraically for t^2 .



Arc-length function

$$\|\vec{r}'(t)\| = \|\langle t, t^2 \rangle\| = t\sqrt{1+t^2}, \quad s(t) = \frac{1}{3}((1+t^2)^{3/2} - 1).$$

Inverse From $(1+t^2)^{3/2} = 3s+1$,

$$t(s) = \sqrt{(3s+1)^{2/3} - 1}.$$

Unit-speed form With $u = \sqrt{(3s+1)^{2/3} - 1}$,

$$\boxed{\vec{R}(s) = \left\langle \frac{1}{2}u^2, \frac{1}{3}u^3 \right\rangle}, \quad s \geq 0.$$

Because s was defined by integrating speed and is strictly increasing for $t > 0$, the chain rule gives $\|d\vec{R}/ds\| = 1$ there; the right-hand unit-speed limit also exists at $s = 0$.