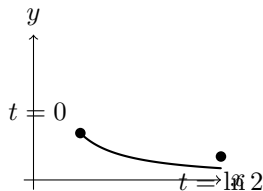


Question 5

Evaluate exactly the length of $\vec{r}(t) = \langle e^t, e^{-t}, \sqrt{2}t \rangle$ for $0 \leq t \leq \ln 2$. The square root in the speed appears complicated; identify its hidden perfect square.

Question 5 – Solution

Strategy Expand the squared speed and recognize $(e^t + e^{-t})^2$.



Speed

$$\|\vec{r}'(t)\|^2 = e^{2t} + e^{-2t} + 2 = (e^t + e^{-t})^2.$$

Both terms are positive, so $\|\vec{r}'(t)\| = e^t + e^{-t}$.

Length

$$L = \int_0^{\ln 2} (e^t + e^{-t}) dt = [e^t - e^{-t}]_0^{\ln 2} = \boxed{\frac{3}{2}}.$$

Insight Simplifying the norm before choosing a substitution is decisive here; treating the radical as generic would hide the elementary antiderivative.