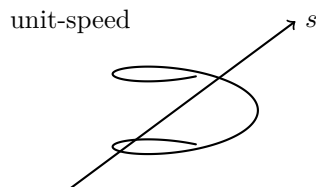


Question 4

Let $\vec{r}(t) = \langle 3 \cos t, 3 \sin t, 4t \rangle$ with starting point at $t = 0$. Find its arc-length function $s(t)$ and reparameterize the helix by arc length. Verify directly that the new parameterization has unit speed.

Question 4 – Solution

Strategy Integrate speed from the stated base point, solve for t , and substitute.



Arc-length coordinate From Question 1, $\|\vec{r}'(t)\| = 5$. Hence $s(t) = \int_0^t 5 \, du = 5t$, so $t = s/5$.

Reparameterization

$$\boxed{\vec{R}(s) = \langle 3 \cos(s/5), 3 \sin(s/5), 4s/5 \rangle}, \quad s \geq 0.$$

Verification

$$\vec{R}'(s) = \left\langle -\frac{3}{5} \sin(s/5), \frac{3}{5} \cos(s/5), \frac{4}{5} \right\rangle, \quad \|\vec{R}'(s)\| = \sqrt{\frac{9}{25} + \frac{16}{25}} = 1.$$

Thus an increase of one unit in s represents exactly one unit of distance along the helix.