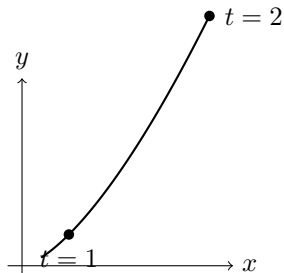


Question 3

For $\vec{r}(t) = \langle t^2, \frac{2}{3}t^3, 0 \rangle$, find the arc length from the point corresponding to $t = 1$ to the point corresponding to $t = 2$. Also find the chord length and compare the two quantities.

Question 3 – Solution

Strategy Use the same speed structure as in Question 2, but retain the nonzero lower limit.



Arc length

$$L = \int_1^2 2t\sqrt{1+t^2} dt = \boxed{\frac{2}{3}(5\sqrt{5} - 2\sqrt{2})}.$$

Chord The endpoints are $P = (1, 2/3, 0)$ and $Q = (4, 16/3, 0)$, so

$$\|Q - P\| = \sqrt{3^2 + \left(\frac{14}{3}\right)^2} = \boxed{\frac{\sqrt{277}}{3}}.$$

Comparison Numerically, $L \approx 5.568$ while the chord is about 5.548. The curve length is slightly larger, as required; equality would occur only for a straight segment traversed without reversal.