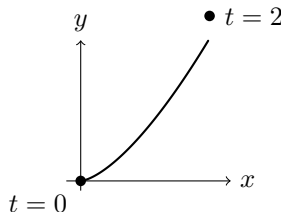


Question 2

Find the exact arc length of $\vec{r}(t) = \langle t^2, \frac{2}{3}t^3 \rangle$ on $0 \leq t \leq 2$. Show the algebraic step that makes the integral elementary.

Question 2 – Solution

Strategy Differentiate first, factor the speed, and use $u = 1 + t^2$.



Speed

$$\vec{r}'(t) = \langle 2t, 2t^2 \rangle, \quad \|\vec{r}'(t)\| = 2t\sqrt{1+t^2},$$

where $t \geq 0$ allows $\sqrt{t^2} = t$.

Integration

$$L = \int_0^2 2t\sqrt{1+t^2} dt = \frac{2}{3} \left[(1+t^2)^{3/2} \right]_0^2 = \boxed{\frac{2}{3}(5\sqrt{5} - 1)}.$$

Check Both coordinate functions increase. The straight-line distance between the endpoints is

$$\sqrt{4^2 + (16/3)^2} = \frac{20}{3}.$$

The computed curve length is larger, as it must be.