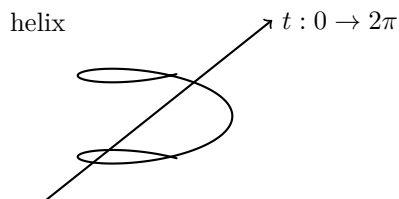


Question 1

A particle follows the helix $\vec{r}(t) = \langle 3 \cos t, 3 \sin t, 4t \rangle$ for $0 \leq t \leq 2\pi$. Find the exact distance traveled. Then explain why the vertical rise alone is not the distance.

Question 1 – Solution

Strategy Distance traveled is the integral of speed, not the net change in any one coordinate.



Speed

$$\vec{r}'(t) = \langle -3 \sin t, 3 \cos t, 4 \rangle, \quad \|\vec{r}'(t)\| = \sqrt{9 \sin^2 t + 9 \cos^2 t + 16} = 5.$$

Arc length Therefore

$$L = \int_0^{2\pi} 5 \, dt = 10\pi.$$

Interpretation The vertical rise is $4(2\pi) = 8\pi$, but the particle simultaneously travels around the cylinder. The velocity has perpendicular horizontal and vertical components of magnitudes 3 and 4, producing total speed 5.