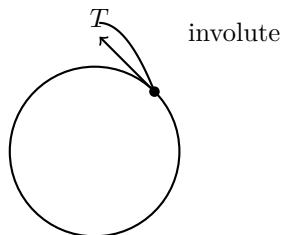


Question 10

For the unit circle $\vec{r}(t) = \langle \cos t, \sin t \rangle$, define its involute by $\vec{q}(t) = \vec{r}(t) - t\mathbf{T}(t)$ for $t > 0$. Find $\mathbf{T}_q(t)$ and show that it is parallel to the circle's principal normal at the corresponding point.

Question 10 – Solution

Strategy Differentiate the involute and use the circle's simple Frenet derivatives.



Circle frame $\mathbf{T}(t) = \langle -\sin t, \cos t \rangle$ and $\mathbf{T}'(t) = \langle -\cos t, -\sin t \rangle = \mathbf{N}(t)$.

Involute derivative Differentiating,

$$\vec{q}' = \vec{r}' - \mathbf{T} - t\mathbf{T}' = \mathbf{T} - \mathbf{T} - t\mathbf{N} = -t\mathbf{N}.$$

Since $t > 0$, $\|\vec{q}'\| = t$, so $\boxed{\mathbf{T}_q = -\mathbf{N} = \langle \cos t, \sin t \rangle}$.

Interpretation The taut string leaves the circle tangentially, while the involute's tangent points along the outward radial direction.