

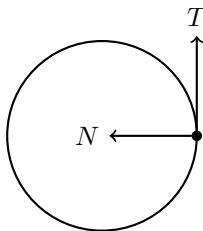
Question 9

The circle $x^2 + y^2 = 4$ is parametrized nonuniformly by $\vec{r}(t) = \langle 2 \cos(t^3), 2 \sin(t^3), 0 \rangle$ for $t > 0$. Find its Frenet frame and explain which parts depend on the varying speed.

Calculus II Section 6-8: Tangent, Normal and Binormal Vectors

Question 9 – Solution

Strategy Let $\theta = t^3$ and separate angular position from the positive rate $d\theta/dt$.



same frame, nonuniform speed

Frame Because $3t^2 > 0$,

$$\mathbf{T} = \langle -\sin(t^3), \cos(t^3), 0 \rangle, \quad \mathbf{N} = \langle -\cos(t^3), -\sin(t^3), 0 \rangle, \quad \mathbf{B} = \langle 0, 0, 1 \rangle.$$

Dependence The frame at a geometric point depends on position and orientation, not on how fast that point is reached. The speed $\|\dot{\mathbf{r}}\| = 6t^2$ varies, but normalization removes it. At $t = 0$ the parametrization stops, so the usual Frenet construction from velocity is not regular there.