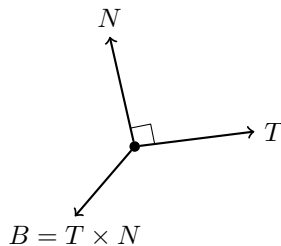


Question 8

Assume $\mathbf{T}$ is a differentiable unit tangent and $\mathbf{T}' \neq \vec{0}$. Prove that the Frenet vectors $\mathbf{T}, \mathbf{N}, \mathbf{B}$ form a right-handed orthonormal basis.

Question 8 – Solution

Strategy Differentiate $\mathbf{T} \cdot \mathbf{T} = 1$, normalize $\mathbf{T}'$, and use cross-product identities.



Orthogonality Differentiation gives $2\mathbf{T} \cdot \mathbf{T}' = 0$. Since $\mathbf{N} = \mathbf{T}'/\|\mathbf{T}'\|$, $\mathbf{T} \perp \mathbf{N}$, and both are unit.

Binormal Define $\mathbf{B} = \mathbf{T} \times \mathbf{N}$. The cross product of perpendicular unit vectors is unit and perpendicular to both.

Orientation By definition $\mathbf{T} \times \mathbf{N} = \mathbf{B}$, so the ordered frame is right-handed. Hence its Gram matrix is the identity and its orientation determinant is $+1$.