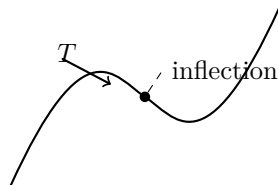


Question 7

For $\vec{r}(t) = \langle t, t^3, 0 \rangle$, determine where the Frenet principal normal is undefined. Does the curve still have a tangent there? Analyze the limiting normals on the two sides.

Question 7 – Solution

Strategy Compute $\mathbf{T}'$ and distinguish loss of curvature from loss of tangent direction.



Tangent $\vec{r}'(t) = \langle 1, 3t^2, 0 \rangle$ never vanishes, so the curve is regular and has tangent $\mathbf{T}(0) = \langle 1, 0, 0 \rangle$.

Normal failure Differentiating the normalized tangent shows $\mathbf{T}'(0) = \vec{0}$, so $\mathbf{N} = \mathbf{T}'/\|\mathbf{T}'\|$ is undefined at 0. For $t > 0$, the normal tends to $\langle 0, 1, 0 \rangle$; for $t < 0$, it tends to $\langle 0, -1, 0 \rangle$.

Interpretation The origin is an inflection point: curvature vanishes and the bending side changes.