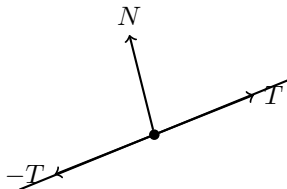


Question 4

A regular curve $\vec{r}(t)$ with nonzero curvature is reparameterized with the opposite orientation by $\vec{q}(u) = \vec{r}(\phi(u))$, where $\phi'(u) < 0$. Determine how $\mathbf{T}, \mathbf{N}, \mathbf{B}$ change. Give a geometric explanation.

Question 4 – Solution

Strategy Apply the chain rule and track the sign introduced by the decreasing parameter.



Tangent Since $\vec{q}' = \vec{r}'(\phi)\phi'$, normalization gives $\mathbf{T}_q = -\mathbf{T}_r$.

Normal Differentiating $\mathbf{T}_q = -\mathbf{T}_r(\phi)$ with respect to u introduces another negative factor ϕ' , so its direction agrees with $\mathbf{N}_r$. Hence $\mathbf{N}_q = \mathbf{N}_r$.

Binormal $\mathbf{B}_q = \mathbf{T}_q \times \mathbf{N}_q = -\mathbf{B}_r$. Thus

$$\boxed{(\mathbf{T}, \mathbf{N}, \mathbf{B}) \mapsto (-\mathbf{T}, \mathbf{N}, -\mathbf{B})}.$$

The bending side is geometric, while forward and right-handed directions reverse.