

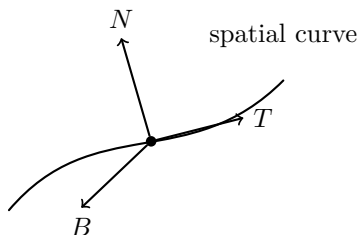
Question 3

Find the Frenet frame for $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ at $t = 1$. Use the cross-product method for $\mathbf{B}$ and verify the orientation.

Calculus II Section 6-8: Tangent, Normal and Binormal Vectors

Question 3 – Solution

Strategy Use $\mathbf{T} = \vec{r}' / \|\vec{r}'\|$, $\mathbf{B} = (\vec{r}' \times \vec{r}'') / \|\vec{r}' \times \vec{r}''\|$, and $\mathbf{N} = \mathbf{B} \times \mathbf{T}$.



Derivatives At $t = 1$, $\vec{r}' = \langle 1, 2, 3 \rangle$ and $\vec{r}'' = \langle 0, 2, 6 \rangle$. Thus

$$\mathbf{T} = \frac{1}{\sqrt{14}} \langle 1, 2, 3 \rangle, \quad \mathbf{B} = \frac{1}{\sqrt{19}} \langle 3, -3, 1 \rangle.$$

Normal Therefore

$$\mathbf{N} = \mathbf{B} \times \mathbf{T} = \frac{1}{\sqrt{266}} \langle -11, -8, 9 \rangle.$$

Verification Direct cross multiplication gives $\mathbf{T} \times \mathbf{N} = \mathbf{B}$ and all norms equal 1.