

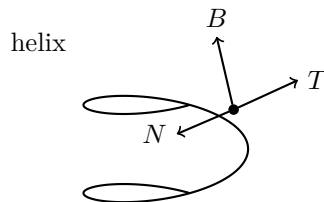
Question 1

For the helix $\vec{r}(t) = \langle \cos t, \sin t, t \rangle$, find the unit tangent $\mathbf{T}$, principal unit normal $\mathbf{N}$, and binormal $\mathbf{B}$ for all t . Evaluate the frame at $t = 0$.

Calculus II Section 6-8: Tangent, Normal and Binormal Vectors

Question 1 – Solution

Strategy Normalize $\vec{r}'$, normalize $\mathbf{T}'$, then use $\mathbf{B} = \mathbf{T} \times \mathbf{N}$.



Frame Since $\|\vec{r}'\| = \sqrt{2}$,

$$\mathbf{T} = \frac{1}{\sqrt{2}} \langle -\sin t, \cos t, 1 \rangle, \quad \mathbf{N} = \langle -\cos t, -\sin t, 0 \rangle,$$

and

$$\mathbf{B} = \frac{1}{\sqrt{2}} \langle \sin t, -\cos t, 1 \rangle.$$

At $t = 0$ $\mathbf{T} = \frac{1}{\sqrt{2}} \langle 0, 1, 1 \rangle, \mathbf{N} = \langle -1, 0, 0 \rangle, \mathbf{B} = \frac{1}{\sqrt{2}} \langle 0, -1, 1 \rangle.$

Verification The three vectors are unit, mutually orthogonal, and $\mathbf{T} \times \mathbf{N} = \mathbf{B}$.