

Question 10

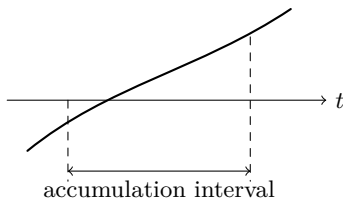
Evaluate

$$\int_0^{\pi} \langle t \sin t, t \cos t, \sin^2 t \rangle dt.$$

Then interpret the result as displacement if the integrand is velocity, and find the average velocity on $[0, \pi]$.

Question 10 – Solution

Strategy Integrate componentwise, using integration by parts for the first two components.



Integral We have

$$\int_0^\pi t \sin t \, dt = \pi, \quad \int_0^\pi t \cos t \, dt = -2, \quad \int_0^\pi \sin^2 t \, dt = \frac{\pi}{2}.$$

Therefore the displacement is $\boxed{\langle \pi, -2, \pi/2 \rangle}$.

Average velocity Divide displacement by the elapsed time π :

$$\boxed{\vec{v}_{\text{avg}} = \langle 1, -2/\pi, 1/2 \rangle}.$$

Verification Multiplying the average velocity by π recovers the net displacement.