

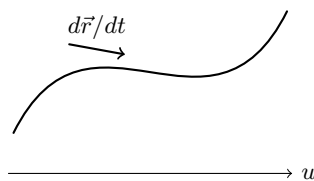
**Question 9**

Let  $\vec{r}(t) = \langle \cos t, \sin t, t \rangle$  and reparameterize by  $t = u^3 - u$ . Find  $d\vec{r}/du$ . Identify every  $u$  where the reparameterized curve momentarily stops, and explain why the geometric helix itself remains smooth there.

## Question 9 – Solution

**Strategy** Apply the vector chain rule and distinguish the image curve from the chosen parameter clock.

speed changes, image fixed



**Chain rule** With  $t = g(u) = u^3 - u$ ,

$$\frac{d\vec{r}}{du} = \vec{r}'(g(u))g'(u) = (3u^2 - 1) \langle -\sin(u^3 - u), \cos(u^3 - u), 1 \rangle.$$

It vanishes when  $3u^2 - 1 = 0$ , so  $\boxed{u = \pm 1/\sqrt{3}}$ .

**Interpretation** The original derivative  $\langle -\sin t, \cos t, 1 \rangle$  never vanishes. The stops occur because  $dt/du = 0$ ; they reflect a nonregular reparameterization, not a cusp in the helix.