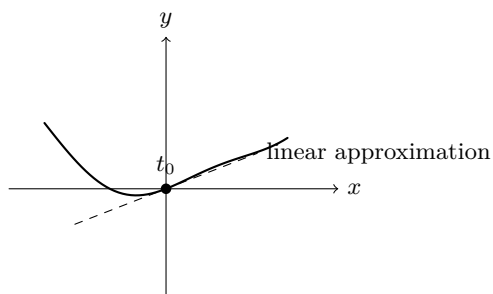


Question 8

For $\vec{r}(t) = \langle \cos t, \sin t, e^t \rangle$, form the second-order Taylor approximation about $t = 0$. Use it to approximate $\vec{r}(0.05)$ and state the order of the neglected error.

Question 8 – Solution

Strategy Use $\vec{r}(0) + t\vec{r}'(0) + \frac{1}{2}t^2\vec{r}''(0)$.



Data $\vec{r}(0) = \langle 1, 0, 1 \rangle$, $\vec{r}'(0) = \langle 0, 1, 1 \rangle$, and $\vec{r}''(0) = \langle -1, 0, 1 \rangle$. Hence

$$\vec{P}_2(t) = \langle 1 - t^2/2, t, 1 + t + t^2/2 \rangle.$$

At $t = 0.05$, $\vec{r}(0.05) \approx \langle 0.99875, 0.05, 1.05125 \rangle$.

Error Because the components have bounded third derivatives near zero, the remainder is $O(t^3)$, here on the scale of 0.05^3 .