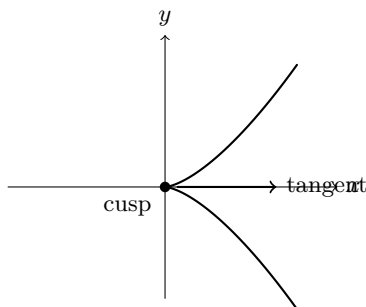


Question 7

Analyze the smoothness and regularity of $\vec{r}(t) = \langle t^2, t^3 \rangle$ near $t = 0$. Does the curve have a tangent line at the origin even though $\vec{r}'(0) = \vec{0}$? Justify using a reparameterization or limiting secant directions.

Question 7 – Solution

Strategy A zero derivative makes the given parametrization nonregular, but the geometric curve may still have a tangent.



Regularity $\vec{r}'(t) = \langle 2t, 3t^2 \rangle$, so $\vec{r}'(0) = \vec{0}$ and this polynomial parametrization is smooth but not regular at 0. Eliminating t gives $y^2 = x^3$, a semicubical cusp.

Tangent direction For $t \neq 0$, the secant direction from the origin is proportional to $\langle t^2, t^3 \rangle = t^2 \langle 1, t \rangle$, which approaches $\langle 1, 0 \rangle$ from both sides. Thus the geometric tangent line exists and is $y = 0$.

Distinction The curve has a tangent at the cusp, but no nonzero velocity there.