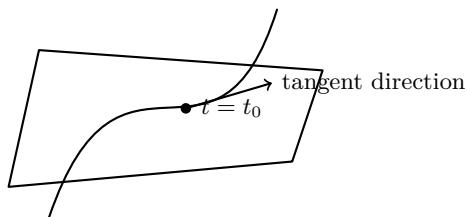


Question 6

For $\vec{r}(t) = \langle t, t^2, t^3 - 3t \rangle$, find all points where the tangent line is parallel to the plane $2x - y + z = 4$. Determine whether any tangent line is perpendicular to that plane.

Question 6 – Solution

Strategy A tangent is parallel to a plane when its direction is orthogonal to the plane normal; it is perpendicular when its direction is parallel to the normal.



Parallel condition $\vec{r}'(t) = \langle 1, 2t, 3t^2 - 3 \rangle$ and $\vec{n} = \langle 2, -1, 1 \rangle$. Thus

$$\vec{r}'(t) \cdot \vec{n} = 3t^2 - 2t - 1 = (3t + 1)(t - 1).$$

So $t = 1$ or $t = -1/3$, giving points $\boxed{(1, 1, -2)}$ and $\boxed{(-1/3, 1/9, 26/27)}$.

Perpendicular condition Parallelism of $\vec{r}'(t)$ with $\vec{n}$ would force scale $1/2$ from the first component, then $2t = -1/2$, but the third component at $t = -1/4$ is not $1/2$. Hence none exist.