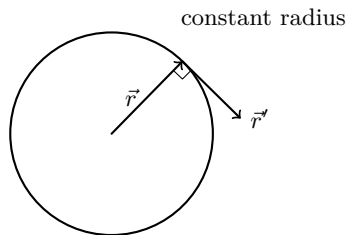


Question 5

Suppose a differentiable vector function satisfies $\|\vec{r}(t)\| = 5$ for every t . Prove that $\vec{r}(t) \perp \vec{r}'(t)$. Apply the result to $\vec{r}(t) = \langle 3 \cos t, 3 \sin t, 4 \rangle$ and verify directly.

Question 5 – Solution

Strategy Differentiate the squared magnitude rather than the square root.



Proof Since $\vec{r} \cdot \vec{r} = 25$, differentiation gives $2\vec{r} \cdot \vec{r}' = 0$. Thus $\boxed{\vec{r} \cdot \vec{r}' = 0}$, proving orthogonality wherever $\vec{r}'$ is nonzero.

Application Here $\vec{r}'(t) = \langle -3 \sin t, 3 \cos t, 0 \rangle$ and

$$\vec{r}' \cdot \vec{r}' = -9 \cos t \sin t + 9 \sin t \cos t = 0.$$

Also $\|\vec{r}'(t)\| = \sqrt{9 + 16} = 5$, as required.