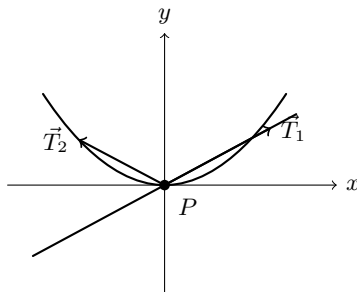


Question 4

The curves $\vec{r}_1(t) = \langle t, t^2, t^3 \rangle$ and $\vec{r}_2(s) = \langle s^2, s, s^3 \rangle$ meet at the origin and at $(1, 1, 1)$. Find the smaller angle between their tangent lines at each intersection.

Question 4 – Solution

Strategy Differentiate each parametrization and evaluate using its own parameter at the shared point.



At the origin Both use parameter 0. The tangent vectors are $\vec{r}_1'(0) = \langle 1, 0, 0 \rangle$ and $\vec{r}_2'(0) = \langle 0, 1, 0 \rangle$, so the angle is $\boxed{90^\circ}$.

At $(1, 1, 1)$ Both use parameter 1. The tangents are $\langle 1, 2, 3 \rangle$ and $\langle 2, 1, 3 \rangle$. Their dot product is 13 and both magnitudes are $\sqrt{14}$, so

$$\boxed{\theta = \cos^{-1}(13/14) \approx 21.8^\circ}.$$