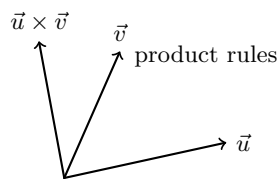


Question 2

Let $\vec{u}(t) = \langle t, t^2, 1 \rangle$ and $\vec{v}(t) = \langle e^t, \cos t, t \rangle$. Compute $\frac{d}{dt}(\vec{u} \cdot \vec{v})$ and $\frac{d}{dt}(\vec{u} \times \vec{v})$ at $t = 0$ using product rules, then verify the dot-product result by expanding first.

Question 2 – Solution

Strategy Use $(\vec{u} \cdot \vec{v})' = \vec{u}' \cdot \vec{v} + \vec{u} \cdot \vec{v}'$ and $(\vec{u} \times \vec{v})' = \vec{u}' \times \vec{v} + \vec{u} \times \vec{v}'$.



Values at zero $\vec{u}(0) = \langle 0, 0, 1 \rangle$, $\vec{u}'(0) = \langle 1, 0, 0 \rangle$, $\vec{v}(0) = \langle 1, 1, 0 \rangle$, and $\vec{v}'(0) = \langle 1, 0, 1 \rangle$.

Results Thus

$$(\vec{u} \cdot \vec{v})'(0) = 1 + 1 = \boxed{2},$$

and

$$(\vec{u} \times \vec{v})'(0) = \langle 0, 0, 1 \rangle + \langle 0, 1, 0 \rangle = \boxed{\langle 0, 1, 1 \rangle}.$$

Expanding $\vec{u} \cdot \vec{v} = te^t + t^2 \cos t + t$ and differentiating also gives 2 at zero.