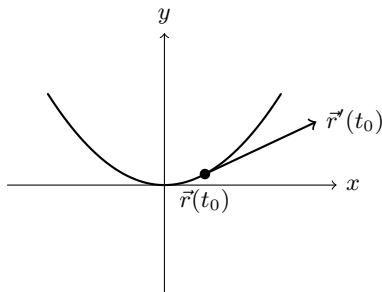


Question 1

For $\vec{r}(t) = \langle t^2 - 1, e^t, \sin(2t) \rangle$, find $\vec{r}'(t)$ and $\vec{r}''(t)$. Find the tangent line at $t = 0$ and a second-order approximation to $\vec{r}(0.1)$.

Question 1 – Solution

Strategy Differentiate componentwise; use the vector Taylor polynomial through second order.



Derivatives $\vec{r}'(t) = \langle 2t, e^t, 2\cos(2t) \rangle$ and $\vec{r}''(t) = \langle 2, e^t, -4\sin(2t) \rangle$. At $t = 0$, $\vec{r}(0) = \langle -1, 1, 0 \rangle$ and $\vec{r}'(0) = \langle 0, 1, 2 \rangle$.

Tangent line $\vec{L}(s) = \langle -1, 1, 0 \rangle + s \langle 0, 1, 2 \rangle$.

Approximation Since $\vec{r}''(0) = \langle 2, 1, 0 \rangle$,

$$\vec{r}(0.1) \approx \vec{r}(0) + 0.1\vec{r}'(0) + \frac{0.1^2}{2}\vec{r}''(0) = \langle -0.99, 1.105, 0.2 \rangle.$$