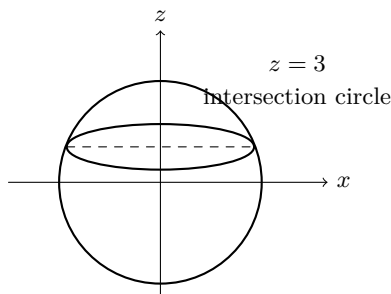


Question 10

Find a parametrization of the curve where the sphere $x^2 + y^2 + z^2 = 25$ meets the plane $z = 3$. Give both orientations and explain why the parametrization covers the entire intersection exactly once on your chosen interval.

Question 10 – Solution

Strategy A horizontal plane cuts the sphere in a circle; determine its radius.



Circle Substituting $z = 3$ gives $x^2 + y^2 = 16$, a circle of radius 4 centered at $(0, 0, 3)$. A counterclockwise parametrization viewed from positive z is

$$\vec{r}(t) = \langle 4 \cos t, 4 \sin t, 3 \rangle, \quad 0 \leq t < 2\pi.$$

The opposite orientation is $\vec{q}(t) = \langle 4 \cos t, -4 \sin t, 3 \rangle$.

Coverage Sine and cosine generate every radius-4 circle point; the half-open interval prevents duplication of the initial point at 2π .