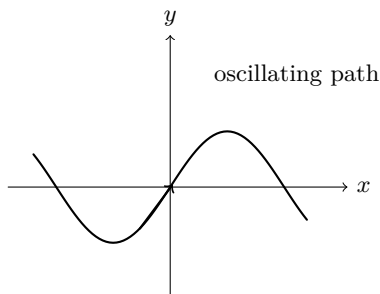


**Question 7**

Analyze the planar vector function  $\vec{r}(t) = \langle t - \sin t, 1 - \cos t \rangle$  for  $0 \leq t \leq 4\pi$ . Find the points for  $t = 0, \pi, 2\pi, 3\pi, 4\pi$ , identify repeated heights, and describe the geometric motion.

## Question 7 – Solution

**Strategy** Evaluate at multiples of  $\pi$  and use the periodicity of sine and cosine.



**Key points** The points are

$$(0,0), (\pi,2), (2\pi,0), (3\pi,2), (4\pi,0).$$

The height oscillates between 0 and 2 while  $x = t - \sin t$  increases from 0 to  $4\pi$ .

**Geometry** The curve is two arches of a cycloid, moving left to right, with cusps at  $(0,0), (2\pi,0), (4\pi,0)$ .

**Verification** Since  $1 - \cos t \in [0, 2]$  and  $x'(t) = 1 - \cos t \geq 0$ , the description is consistent.