

Question 6

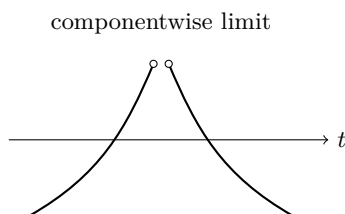
Find constants a, b, c so that

$$\lim_{t \rightarrow 0} \left\langle \frac{\sqrt{1+at}-1}{t}, \frac{\sin(bt)}{t}, \frac{e^{ct}-1}{t} \right\rangle = \langle 2, 3, -4 \rangle.$$

Justify the limit component by component.

Question 6 – Solution

Strategy A vector limit equals the target precisely when all component limits equal their corresponding target components.



First component Rationalizing gives

$$\frac{\sqrt{1+at}-1}{t} = \frac{a}{\sqrt{1+at}+1} \rightarrow \frac{a}{2},$$

so $a = 4$.

Other components Standard limits give $\sin(bt)/t \rightarrow b$ and $(e^{ct}-1)/t \rightarrow c$. Hence $a = 4, b = 3, c = -4$.

Verification The three component limits are respectively $2, 3, -4$.