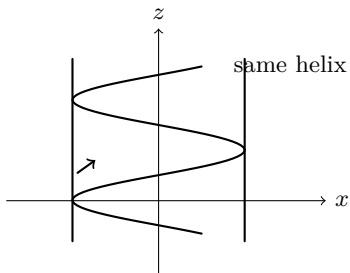


Question 4

Show that $\vec{r}_1(t) = \langle \cos t, \sin t, t \rangle$ and $\vec{r}_2(u) = \langle \cos(2u), \sin(2u), 2u \rangle$ have the same image. Compare their orientations and speeds of traversal at matching points. Then give a reparameterization with the opposite orientation.

Question 4 – Solution

Strategy Relate parameters by matching all three components.



Same image Setting $t = 2u$ makes $\vec{r}_1(t) = \vec{r}_2(u)$, so both trace the same helix. As each parameter increases, z increases, so orientations agree.

Traversal rate A change Δu produces twice the angular and vertical change of the same Δt ; equivalently, $\vec{r}_2(u) = \vec{r}_1(2u)$ traverses the image twice as fast with respect to its parameter.

Reverse orientation For example, $\boxed{\vec{q}(s) = \langle \cos s, -\sin s, -s \rangle = \vec{r}_1(-s)}$.