

Question 1

Find the domain of

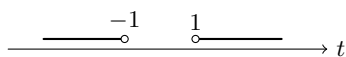
$$\vec{r}(t) = \left\langle \sqrt{t+2}, \ln(1-t), \frac{1}{t^2-1} \right\rangle.$$

Evaluate $\vec{r}(-1/2)$ and determine whether $\lim_{t \rightarrow -1^+} \vec{r}(t)$ exists in $\mathbb{R}^3$.

Question 1 – Solution

Strategy Intersect the domains of all three scalar components; vector limits exist exactly when every component limit is finite.

common component domain



Domain The restrictions are $t \geq -2$, $t < 1$, and $t \neq \pm 1$. Hence

$$\boxed{[-2, -1) \cup (-1, 1]}.$$

Also $\vec{r}(-1/2) = \langle \sqrt{3/2}, \ln(3/2), -4/3 \rangle$.

Limit As $t \rightarrow -1^+$, $1/(t^2 - 1) \rightarrow -\infty$, so no finite vector limit exists in $\mathbb{R}^3$.