

Question 7

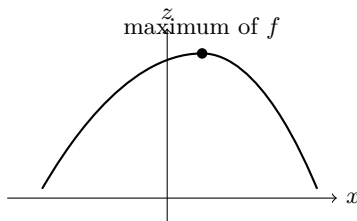
Without using derivatives, find the range and all global extrema of

$$f(x, y) = 7 - 2x^2 - 2xy - y^2 + 8x + 6y.$$

Hint: rewrite the quadratic as a sum of squares.

Question 7 – Solution

Strategy Complete squares in a way that respects the mixed term.



Rewrite Since $2x^2 + 2xy + y^2 = (x + y)^2 + x^2$,

$$f = 7 - (x + y)^2 - x^2 + 8x + 6y.$$

Set $u = x + y$. Then $y = u - x$ and $f = 7 - u^2 - x^2 + 2x + 6u = 17 - (x - 1)^2 - (u - 3)^2$.

Conclusion Therefore

$$\boxed{f(x, y) = 17 - (x - 1)^2 - (x + y - 3)^2}.$$

The global maximum is 17 at $x = 1$, $x + y = 3$, hence $(1, 2)$. There is no minimum, and the range is $(-\infty, 17]$.