

Question 6

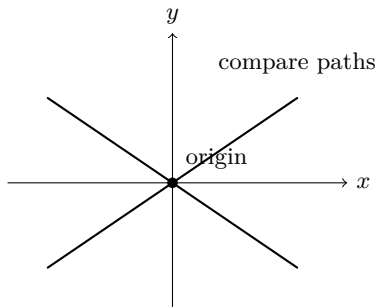
Investigate

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2}.$$

Evaluate along at least three families of paths and explain why the evidence is decisive.

Question 6 – Solution

Strategy A multivariable limit must have the same value along every path.



Path tests Along $y = 0$, the expression equals 1; along $x = 0$, it equals -1 ; and along $y = mx$, it equals

$$\frac{1 - m^2}{1 + m^2},$$

which varies continuously with m .

Conclusion Since even two paths give different values, the limit does not exist.

Why decisive Path agreement can only suggest a limit, but one disagreement proves nonexistence because the definition requires uniform behavior over all approaches.