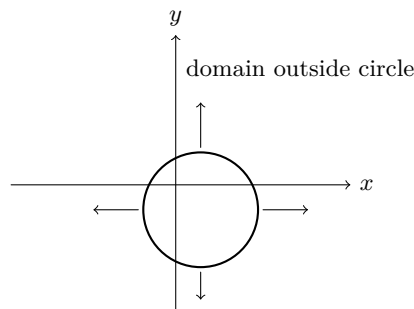


Question 5

Let $f(u, v) = \sqrt{u - v}$ and $g(x, y) = (x^2 + y, x - y^2)$. Find $(f \circ g)(x, y)$ and its domain. Determine the boundary and whether the point $(1, 1)$ is interior, exterior, or on the boundary.

Question 5 – Solution

Strategy Substitute both components of g into f and require a nonnegative radicand.



Composition We obtain

$$(f \circ g)(x, y) = \sqrt{x^2 + y - x + y^2}.$$

Thus the domain is $x^2 - x + y^2 + y \geq 0$, or

$$\boxed{(x - 1/2)^2 + (y + 1/2)^2 \geq 1/2}.$$

It is the exterior of a disk, including its circular boundary.

Point test At $(1, 1)$ the original radicand is $2 > 0$, so the point is interior to the domain.